EFFECTIVENESS OF INSTRUCTION FOR TEACHERS OF ELEMENTARY SCHOOL MATHEMATICS IN LARGE GROUPS WITH SMALL DISCUSSION GROUPS

By

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CHAPTER I

INTRODUCTION

More profound changes in mathematics and mathematics instruction have occurred in this century than have ever occurred in the history of the subject. These changes are not restricted to any one level of education but are taking place from kindergarten through graduate schools. Never has more emphasis been placed on the teaching of and the scope and sequence of mathematics than at the present time. Great interest in the new mathematics is being expressed by the public as well as by the educators.

The rapid changes taking place in the mathematics curriculum of the public schools reflect many of the scientific and technological advances taking place in our society. These changes in the mathematics curriculum cannot accurately be described as purely mathematical since they reflect social, educational, psychological, and mathematical advances.

There is unanimous agreement that the social situation has changed drastically in the past one hundred years. During the early part of the nineteenth century comparatively few people were engaged in research. In general, the public needed to know the four fundamental operations of arithmetic and basic measurement procedures. However, by the end of the nineteenth century new social needs were arising and new mathematics was being developed to meet those needs. New fields of mathematics seemed to spring up overnight. These were fields such as linear programming, game theory, topology, and sampling theory. It was stated that:

"More mathematics . . . has been created during the first half of this century than during all the rest of history" (13:1). According to a report by the Mathematical Association of America, "There are more research mathematicians alive today than the total number in the several thousand-year history of the subject" (11:3).

The rapid introduction into our society of automation, high-speed computers, advancing technology, and fundamental research in all fields are some of the basic causes for the rapid changes taking place in mathematics. Now, more than ever before, mathematics has become an integral part of all scientific fields. It is at present almost impossible to do advanced work in any scientific field without a knowledge of some area of mathematics. There is every indication that even more profound changes in the demand for mathematics and in the nature of mathematics itself will occur in the next decade. For example, computer mathematics may become a prominent part of the mathematics curriculum. Experimental work with this type of content is being carried on at present. Walter Hoffman of Wayne State University states:

> I have been teaching computer programming to high school teachers as well as to high school students for the past six or seven years. My last experience, however, as an experiment, involved teaching computer programming to 11 gifted children of various grade levels ranging from the fourth to the sixth grade. We met for eight one-and-one half hour sessions at intervals of one week. These gifted children quickly became excited about the use of a computer and learned how to program it in Fortran. During the short duration of this experiment, the students acquired the ability to program solutions of specific quadratic equations with positive integral roots, without any knowledge of the nature of quadratic equations in general. The only conclusion I wish to draw from this experiment is that we do not know the earliest possible educational or age level at which children can be successfully taught to program computers.

> Accordingly, some research is called for to determine the optimum initial level at which this subject should be taught. (14:19)

Basically, the modern mathematics program differs from the traditional program in the following ways:

1. About 20 per cent of the content is new, or at least did not appear in the traditional program.

2. Some of the older topics have been deleted and others de-emphasized to make room for new topics which are in the stream of modern mathematical thought.

3. The approach to mathematics has shifted from just the teaching of manipulation to the teaching of "how" and "why"--involving both skills and understanding. Thus, the logical structure of mathematics is emphasized.

4. The discovery method of teaching is emphasized. It is assumed this will make both the teaching and learning of mathematics more interesting and rewarding.

5. New terminology is introduced to unify mathematical concepts.

6. A method of inquiry, process of thought, is emphasized.

7. Application of mathematics is de-emphasized. This particular change is rather controversial as there is not general consensus favoring the change.

Now may attention be specifically directed to the changes being called for in the elementary schools' mathematics curriculum. Clyde G. Corle, of Pennsylvania State University, appropriately states four basic changes in elementary school mathematics:

> First of all, mathematics for small children is viewed as a combination of several mathematical sciences, each contributing in simple ways to children's competency with numbers. Memorization of meaningless number facts has been replaced by reasoning, by the study of principles, postulates, and logic. . . . The structure of the number system is an important part of modern mathematics and young children are taught understanding of it, including the decimal system of notation, through work with other number bases.
>
> The second change which the new mathematics has brought about is a more careful use of quantitative vocabulary. Mathematicians have developed terms which carry definite meanings, and they use words with great care. Many elementary teachers, however, over-simplify numerical ideas and have taught children to use terms which are inadequate, inaccurate, and sometimes just plain ridiculous. . . .
>
> The third change advocated by modern mathematicians is that of increased emphasis upon understanding the computational operations. Computation has long been a process of following mechanical formulas, one of proceeding step by step in search for some desirable result, called "The Answer." The "steps" are often completely detached from meaning, and the procedures depend almost totally upon memorized responses. . .
>
> The fourth change which is supported by the "new mathematics" is that of giving the responsibility for learning back to the children. If mathematics is going to be more than a set of procedures, children must learn in their own way to understand the principles upon which these procedures have been established. While nobody would wish to discredit the lecture-demonstration technique of teaching, its use to the exclusion of other methods does not invite children to do creative thinking. . . . Modern mathematics revolves around the discovery method of teaching. (2:242)

These changes have created a crisis for our schools. Immediate efforts must be made to revise and incorporate new topics and new approaches into the curriculum in order to afford the students a mathematics education appropriate for our times. This study deals with the present problem of updating the mathematics competency of large numbers of teachers. The study focuses specifically on teachers of elementary school mathematics.

Need for the Study

One of the necessary conditions for an adequate mathematics program is qualified teachers. A well-qualified teacher must know his subject area and teach with enthusiasm and interest. A teacher who fears or dislikes mathematics probably will not teach much mathematics to his students but will readily convey his fears and his dislike of the subject to his students. One of the best ways to attract students to mathematics is for the instructor to know and like mathematics and to communicate his knowledge and interest to others.

Most teachers of mathematics are now facing the need for re-education because of the changes that have recently taken place in the subject area itself. Their original formal education was received before many of these significant changes in mathematics came about. Teachers are now faced with using unfamiliar approaches and teaching new content in the modern mathematics programs. Extensive in-service re-education is under way over the entire nation. The need is very great. If the present demands for in-service education are met within any practical length of time, large-scale instruction becomes a necessity.

This conclusion becomes apparent when we examine several factors. First is the matter of time. In a large county in Florida, if one course in modern mathematics were offered each trimester with a maximum enrollment of thirty-five teachers in each class, it would take more than ten years to reach all of the elementary teachers in this county. During this time, further great changes would be taking place. One would therefore get farther behind than he was at the start.

The second factor is the amount of instruction. One course in modern mathematics would not meet the needs or demands even at the present time. A sequence of in-service courses for elementary school teachers should be envisioned. Any single course is but a part of the answer. The goals set by various writers all call for more than one course. For example, the American Mathematical Association submitted the following recommendations in 1961:

> As a prerequisite for the college training of elementary school teachers, we recommend at least two years of college preparatory mathematics, consisting of a year of algebra and a year of geometry, or the same material in integrated courses. It must also be assured that these teachers are competent in the basic techniques of arithmetic. The exact length of the training program will depend on the strength of their preparation. For their college training, we recommend the equivalent of the following courses:
>
> a. A two-course sequence devoted to the structure of the real number system and its subsystems.
> b. A course devoted to the basic concepts of algebra.
> c. A course in informal geometry. (11:11)

Jack N. Sparks (21), Colorado State College, summarized the recommendations of several leading mathematics educators with regard to the arithmetic understandings that elementary school teachers need. A careful consideration of these recommendations leads to the conclusion that not one of these recommendations could be satisfied within the scope of a single course. Hence even if a single course in modern mathematics could reach a large number of teachers in a practical period of time, the need for in-service education would not have been satisfied.

A third factor is a shortage of qualified leaders to provide the needed in-service instruction. There is a great demand for competent mathematics educators to work with in-service teachers. This shortage has forced many universities and public school systems to examine the possibilities of large-group instruction. However, if enough qualified leaders were available, assuming normal-size classes would be conducted, it would not be practical to employ a sufficient number. The cost, due to the large number of mathematics educators needed, would be astounding and in most cases prohibitive. Thus the element of time, the number of needed courses, and the lack of qualified staff are major factors which call for the use of other methods of instruction such as large-group instruction, television, programmed materials or faculty study under the guidance of local secondary mathematics teachers. This study will deal with the effectiveness of a plan for large-group instruction.

Any proposal, however, to use large-group instruction, team-teaching, television, organized faculty study, or any other scheme, must be critically examined to see if it is in keeping with the quality of education we desire. In evaluating the merit of any one of these procedures, one must recognize that there are many outcomes of teaching, and that a method should not be evaluated solely on the basis of the knowledge outcome. Certainly one major concern is the direct cognitive knowledge of mathematics, but teachers must also develop essential psychological understandings.

During the summer of 1961, a conference was held at the University of California, attended by some of the outstanding mathematics educators in America. Davis (3) stated that one of the areas of concern of these educators was the matter of negative attitudes on the part of many teachers toward mathematics. They stressed that teachers must understand and enjoy mathematics if they are to inspire pupils to do likewise. In the plan for the present study, therefore, an assessment of changes in the teachers' attitudes toward mathematics was provided for, in addition to measuring their understanding.

Statement of the Problem

Will the effectiveness of large-group instruction, followed by sub-group discussion and demonstration teaching, be comparable to that of classes of regular size? More particularly, do teachers learn well under the conditions of this experiment? Do they acquire the essential psychological understandings? A course was designed for two hundred twenty-five elementary school teachers and the results were compared with instruction in a number of regular-sized classes. This study was experimental in nature. Certain basic steps in the study were:

1. An instrument to test for understanding of basic concepts in elementary mathematics was used as a pretest and posttest in both the large and regular size classes.

2. A survey of the teachers' attitudes toward arithmetic was made at the beginning and end of the course in each of the classes involved.

3. The students (elementary school teachers) were requested to give a personal evaluation of the effectiveness and desirability of the instruction provided. An evaluation instrument was devised for this purpose.

A full description of the procedure used in carrying through the experiment, together with a description of the instruments used, will be given in Chapter III and IV.

Meanwhile, Chapter II will review the results of other selected studies dealing with ideas relevant to conducting this experiment.

CHAPTER II

A REVIEW OF RELATED RESEARCH

Previous research will be examined to ascertain its findings with regard to the question: Can instruction in mathematics for large groups of students be as effective as instruction for regular size classes?

In addition to the principal question of this study, there are a number of closely related questions of considerable interest. Does the teacher who fears or dislikes mathematics pass on his fears and dislikes to the student? Will such a teacher develop a more favorable attitude as he gains mathematical understandings? Will the students' attitude toward the subject have a definite effect on their achievement? Will it also influence his choice of vocation? Selected research studies will be examined concerning the significance of both the teacher's and the students' attitudes toward mathematics.

Thus, the present review will be concerned with two broad but overlapping areas: first, research regarding the effectiveness of large-group instruction, and second, research regarding attitudes toward arithmetic or mathematics.

Research Related to Large-Group Instruction

We may now direct our attention to certain research related to the effectiveness of large-group instruction. A large class is interpreted to be a class which exceeds an enrollment of thirty-five students. Actually, the writer is interested in classes which have an enrollment of more than forty and less than four hundred. A small or regular size class is regarded as one which has an enrollment of approximately thirty-five students or less.

The effectiveness of large-group instruction is often considered only in terms of student achievement. However, in the present study, the writer will also consider change in attitudes, desire to do more advanced work, and opinions as to the relative value of the course.

A historical survey of research literature failed to give a "magic number" for the optimum class size. Alvin C. Eurich gives this account of the assumed twenty-five students for each teacher:

> . . . where did we get our fixed notion of one teacher for every twenty-five students? I have searched long. With a clue from a casual conversation and with the aid of an eminent Talmudic scholar, President Samuel Belkin of Yeshiva University, I now find the answer goes back to at least the middle of the third century. In Babylonian Talmud Baba Bathra 21 A the rule was established by Rabbi Raba, an authoritive sage of his era: "Twenty-five students are to be enrolled in one class. If there are from twenty-five to forty an assistant must be obtained. Above forty, two teachers are to be engaged. (7:240)

Eurich carefully points out that this rule was stated long before books; to say nothing of modern means of communication such as: telegraphy, photography, motion pictures, phonographs, microphones, radio, tape recorders, television, and modernophones. The writer would also like to include recent developments in programmed materials and teaching machines. With all these improvements in communication, is it not strange we should still be arguing today--in the twentieth century--to preserve a third-century formula?

This issue is still far from settled. As John T. Warbuton points out:

> The heavily endowed philanthropic corporations and the Federal government both have contributed money toward well-controlled experimentation in varied class sizes. However, most of the money was spent on gadgetry and not on basic experimentation designed to determine the relative teaching effectiveness in different size groups. In spite of a great emphasis placed on these new horizons in education through the Trump report and other equally important studies, a thorough search of the literature revealed a scarcity of articles dealing with experiments designed to show optimum size of classes for teaching. (24:430)

Warbuton notes that vast breakthroughs in the electronic field have provided education with new tools which make groups of various sizes not only a possibility but a reality. Eurich states that a review in the Encyclopedia of Educational Research of more than fifty years of investigation states:

> . . . On the basis of criteria used in the experiment studies published to date and under typical group teaching procedures, mere size of class has little significant influence on educational efficiency as measured by achievement in the academic subjects.
>
> Although experimental evidence does not provide a clear-cut answer to the class-size issue, the general trend of the evidence places the burden of proof squarely upon the proponents of small class. (7:241)

It is obvious, as research literature regarding class size is examined, that substantial evidence is not available supporting an optimum class size. However, there is little or no evidence that decisions have deliberately been made to have large classes, since in most cases circumstances have forced such decisions. Various types of situations have developed so that large-group instruction has become a necessity. Eurich (7:239) points out that many institutions have found great student demand for certain exceptional professors. The demands became so great that it was impossible to restrict their classes to small numbers.

The great crisis of the present time is to educate an ever more rapidly increasing student population. Most colleges and universities are now experiencing an overwhelming enrollment. Experimentation in large-class instruction is increasing at many of these institutions. Frank Ray Wilkinson (25), in a research survey, gives a bibliography of seventy-nine articles relating to this general problem. Wilkinson offers an outline that may be used as a guide in making the proper evaluation of the whole problem of class size and teaching method.

An article in the Harvard Education Review states:

> The anticipated increase in the number of students applying for admission to the nation's colleges is a point that need not be labored here. The pressure of enrollment is already beginning to mount in some quarters, leading to renewed concern about the feasibility of instituting or revising procedures for selecting students. In addition, institutions are becoming concerned about procedures that will increase instructional efficiency while, at the same time, maintain or improve the quality of the educational experience. The solution to the general problems created by large numbers of applicants in the face of plant and staff deficiencies must, of course, be a many-faceted one. The present paper summarizes research findings bearing upon just one of several possible solutions: i.e., that of increasing class size. (19:216)

This is exactly the type of situation which gave impetus to the present study. It is a situation where large numbers of teachers demanded in-service work in the modern mathematics programs. If class size were restricted to twenty-five it would be impossible to reach even a fractional part of the teachers in a practical length of time. Hence, large-group instruction was regarded as a possible solution.

University of Oklahoma Study

An experiment in large-group instruction was conducted at the University of Oklahoma during the school year 1947-1948. Three instructors took part. Instructor A had a large and small class, Instructor B had a large and a small class, and Instructor C had two small classes. The two large classes slightly exceeded three hundred students and the small classes ranged from twenty-three to thirty-one students. The same material was covered by both instructors and each gave identically the same assignments with the same number of examinations of equal length. The course was American Government.

The attitudes of the students were measured by using a modification of an attitude scale originally developed by Sartain and Waring. This scale was administered twice during the semester. Marks were released before mid-term and the final week of the semester. No significant differences were found in the attitude-expression results regarding the responses to the question: "How difficult do you feel the course to be?" Also there were no significant differences in the relative amount of interest expressed in the course.

One of the major findings of the study according to the author was:

> The most significant finding of this study is that the amount of achievement, as measured by standardized tests, and the attitudes of the students toward American Government, varied as a function of the course instructor and did not vary as a function of size of class. This suggests that the differential skills and abilities of the instructors to present materials to large and small classes is the critical variable. No statistically significant differences were observed between the small classes taught by the lecture or discussion methods, but differences were revealed in the achievement of students

> when taught by different instructors. There was interaction between the teacher and the size of class on the felt difficulty of the course and the interest aroused in the course. These latter differences are interpreted as being a function of the instructor rather than the size of the class. (18:279)

This study goes on to conclude that any interpretation of the results should consider certain characteristics of the particular course taught. The course was introductory rather than advanced, and required learning of general principles and processes rather than skill in manipulation of these principles and processes.

Miami University Study

Miami University of Ohio conducted an experiment using three general approaches to large-group instruction. This experiment was during the 1957-1958 academic year. The findings of this study are reported in the 1958-1959 volume of the Harvard Education Review. The following is a summary of this study.

The three general approaches were: (a) large lecture sections with relatively little discussion in class by the students; (b) large section taught by a combination of lecture and problem-solving or case-study techniques; (c) multiple sections of approximately thirty-five students, each receiving simultaneous instruction by closed-circuit television. The classes using the three general approaches listed above were referred to as experimental sections. Eleven courses were studied during this experiment with each course having an experimental and a control section. The students in the control and experimental section were routinely equated on the basis of tests of proficiency in mathematics and English as well as tests of academic ability.

The large lecture classes--approach (a)--ranged from forty-three to one hundred seventy-two students in each class. The lecture and problem solving--approach (b)--ranged from sixty-five to seventy students in each class. The content of each course, final examination, and instructor were constants in the control and experimental section of each course. The courses taught were: Air Science, Educational Psychology, Zoology, Chemistry, French, Psychology, Government, Freshman Composition, Geography, and History.

In the evaluation of the effectiveness of these courses, primary interest focused on achievement, students' attitudes regarding the course and instructor, and the students' attitudes regarding the mode of instruction. In a discussion of the findings of the study, the researchers found that apparently large-group instruction does not have deleterious effects upon the achievement as measured within the framework of the study. However it must be pointed out that achievement has been restricted in scope to immediate and not long-range retention. It is also significant to point out the possible defects in the criterion instruments that propose to measure achievement. The study concludes that additional research, based on the use of unique measures stressing other kinds of achievement than mere retention of facts, is direly needed.

Students' attitudes about the course and instructor indicated that the small sections were rated slightly higher than the large experimental sections. The students' attitudes about the mode of presentation also indicated that the students, in general prefer instruction in small classes to instruction in large classes. This was regardless of whether the small class was taught by television, lecture approach, or case-and-problem approach. The researchers offered the following explanation regarding student preferences:

> The finding that students, in general, prefer conventional instruction to large-group instruction is not particularly surprising. The expectations about instructional procedure developed in the primary and secondary schools predispose students to this preference. It will be interesting to note the effects, if any, of large-group instruction (primarily televised) currently being received by some grade-school and secondary pupils upon their subsequent expectations as university students. In spite of the bias for conventional instruction still prevalent at some institutions, at the present time there is an indication that large-group instruction is more palatable to college students when they are given an opportunity to participate than when they are not given such an opportunity. . . . it must be noted also that the instructor is probably the most important determinant of student attitude toward the mode of instruction. Variations in attitude between courses seem to reflect much more the student attitude toward the effectiveness of the instructor than they do the attitudes toward the mode of instruction per se. (19:226)

Orange Coast College Study

The Orange Coast College, Costa Mesa, California, began plans for an experiment in large-group instruction in the summer of 1957. The experiment was conducted for several semesters with two hundred and fifty students in each class. The instructor was given ample time to plan and work with the class. He was given an assistant and visual aids were made available. The students would meet for a large-group lecture two hours each week and one hour each week the students would meet in small groups of twenty-five to thirty students. The course taught was United States History.

An evaluation was made at the end of each large class which resulted in some general conclusions:

> 1. Student reaction was overwhelmingly in favor of this "large" as opposed to the usual "small" class.
> 2. More than 80 percent of the students enrolled in large classes stated they learned as much or more than they assumed was learned in small classes.
> 3. The drop out rate of students in large and small classes was approximately the same.
> 4. More than 90 percent of the students enrolled in the large class experiment indicated they would recommend the large class to fellow students.
> 5. Less than 5 percent of those enrolled for the large class experiment chose to enroll for the small class the following semester in order to complete the second half-year of the course.
> 6. A standardized United States History examination given to the large history class and to all classes of the standard small type at the end of the semester showed that students as a group from the large class scored slightly higher than those from small groups. The difference in performance was not statistically significant. (15:74)

The results of this study were so encouraging that a large lecture demonstration was built especially designed for effective large-group instruction. According to this research study, an exceptional teacher is a necessity in order to promote superior instruction. This instructor, as described, must be imaginative, an organizer, a dynamic speaker and one who can motivate students. This research study also contends that large-group instruction leads to superior teaching since the teacher must be better prepared and the program is enriched through the use of many visual aids. A higher level of instruction is promoted by teacher cooperation.

Fordham College Study

A study was made at Fordham College, New York City, during the academic year 1956-1957. The purpose of the study was to determine if a well-qualified teacher may effectually teach classes considerably larger than the traditional thirty students. The courses taught were: Principles of Economics for freshmen, Introduction to American Government, and Introductory Sociology. The latter two courses were for sophomores. The large classes contained sixty students and the control sections contained thirty students. The study paired one large with one control class and the same professor, using the same syllabus, taught each pair of test sections. The author of the study offers this partial summary of the results and conclusions:

> Summarily, statistical examination of the data derived from the experiment supports its basic hypothesis that a large class with good quality will equal the achievement of a small class with the same quality. This proved true with reference not only to the academic subject matter, but also to the "intangible" purposes of the basic social science courses as these purposes were viewed by the experimentors--social awareness and principled synthesis of social outlook. Of the twelve mean measures analyzed, only one (social outlook for Sociology) significantly favored the small group. In the other 11 cases, the small group surpassed its respective large group five times; the large surpassed the small five times; and the groups were identical once. (1:340)

The basic criticism of large-group instruction offered by participating professors was of the greater effort needed to establish classroom informality. Greater difficulty was also experienced in uncovering students' individual academic difficulties and in stimulating the complacent members of the class.

University of Wichita Study

An experiment in large-group instruction was conducted at the University of Wichita in 1956 and 1957. The course taught was Intermediate

Algebra. Three large classes were conducted with an average enrollment of approximately eighty-seven students. The same professor taught each large class. There were twelve different professors for the fourteen small sections conducted. The average small class enrollment was about twenty-one students.

The researchers of this experiment offered this concluding statement:

> This experiment was a cooperative effort on the part of both teachers and administrators to find a solution to the problem of increasing enrollment and a limited t teaching staff. However, the results reported here indicate that the course in intermediate algebra, if it is taught at all, should be taught in small discussion classes. It is felt that the poor showing in the large lecture classes may have been due to the low initial ability-level of all the students who enrolled in this remedial course in algebra. It is hoped that similar studies may be undertaken using students enrolled in the regular freshman courses in mathematics. If such further experiments are made, the numerous conference sessions used in this study can hardly be justified. With the prospects of a continuing inbalance between the number of students enrolled in college level mathematics and the number of qualified instructors, every effort should be made to discover acceptable methods of teaching more students with the teaching staff now made available. (20:315)

Hence, we find some studies in this area which have not experienced success. It has been a deliberate effort of the writer to give an overall picture of the large group teaching situation by reporting both favorable and unfavorable experiments.

Research Regarding Attitudes Toward Mathematics

Recent research tends to confirm the assumption that the students' attitudes toward mathematics play a vital part in their learning. Herbert F. Spitzer of the State University of Iowa states:

> The measures used in research show the relationship between favorable attitudes and achievement, although low, is clearly positive. The creation and maintenance of a favorable attitude toward arithmetic should be, therefore, a major concern of the classroom teacher. (22:11)

In 1955, Thomas Poffenberger and Donald A. Norton (16) (17) began a study regarding factors determining attitudes toward arithmetic and mathematics. This study was initiated because of the alarming shortage of qualified personnel in the scientific fields such as engineering, physics, chemistry, and mathematics. Since each of these scientific areas requires a knowledge of mathematics, the authors assumed that the students' attitude toward mathematics would affect their choice of occupation. Thus, if the student possessed a negative attitude toward mathematics, he would not choose any occupation requiring a thorough knowledge of an area of mathematics. They published the conclusions of their preliminary survey in 1956. The authors' preliminary study listed three major factors which possibly have strong effects on students' attitudes toward mathematics. One of the factors listed was the teacher. This preliminary survey shows the importance of the teacher's role in determining the attitude of students. Poffenberger and Norton continued their research in this area and in 1959 published their final conclusion. The conclusions of the second study tended to support and confirm the results of their preliminary study. In every case, the classroom teacher is listed as a major factor in influencing students' attitudes.

This conclusion is also supported by C. R. Mill (12). Mill states that a student's attitude toward his subject area can hamper or greatly help in the learning process. He concludes that the teacher is instrumental in influencing the student's attitude.

One of the major conclusions reached in a study by Harold H. Lerch was:

> The student's success in arithmetic and his attitude toward arithmetic are more basically dependent upon his teachers' attitudes and the methods they employ. Thus, classroom organization procedures and teaching methods should be the ones which assure each pupil a measure of success at his own level of ability and understanding and which at the same time encourage development and maintenance of favorable attitudes. (10:119)

The student's attitude is, again, considered highly significant by Joseph Stipanowich (23). He points out that, in spite of a large number of improvements in instruction, arithmetic is taught so badly that students rarely study any branch of mathematics beyond minimum requirements. Stepanowich believes that because of the teachers' lack of understanding of mathematics, they are unable to interpret and implement at the classroom level. Thus, they themselves communicate their own inadequacies to the students through their attitudes of insecurity in the subject.

As these research studies are examined, it becomes apparent that attitudes play a vital part in the learning process. This is true not only for students at the elementary school level but for all levels. Every major research study examined listed the classroom teacher as a major factor in influencing students' attitudes. Most researchers in this area agree that a teacher with a negative attitude toward mathematics will, in general, influence students to likewise dislike mathematics. It has also been pointed out that in most cases a teacher's negative attitude is apparently related to his lack of understanding of the subject. If this is the case, then as a teacher becomes more competent in mathematical understanding, he should develop a more favorable attitude.

Spitzer (22) states that while all authorities who have written on the subject agree that a good mathematical background is a necessary part of a teacher's preparation, there is no significant evidence that typical college mathematics courses actually contribute to superior teaching of arithmetic. It is now conceivable that effective mathematics courses for teacher preparation must take into account the psychological outlook of the teacher toward the subject. Mathematical understanding and application must be emphasized. This is in contrast to rote skills and meaningless memorization.

Many efforts have been made in the past to measure both teachers' and students' attitudes toward mathematics. In 1951, Wilbur H. Dutton (5) (6) of the University of California at Los Angeles prepared a measuring scale for this purpose. This scale is used diagnostically to show both favorable and unfavorable attitudes of elementary school teachers toward arithmetic. This scale will be used in the present study in an attempt to detect changes in the teachers' attitude. This instrument, and how it will be used, will be explained in the chapter regarding instruments.

Summary

There is every indication that the student-teacher ratio will go up in the near future. Hence, there is great need for all types of experimentation in teaching large groups of students. Studies are now being conducted at the public school as well as the college level. However, the effectiveness of many of these studies is considered only in terms of achievement. As pointed out earlier in this paper, effectiveness should also be considered in terms of the psychological outlook of the student. Every consideration should be given to using instruments in an effort to measure students' attitudes as well as achievement.

Many research studies regarding large-group instruction and attitudes were available to the writer. However, all of these studies could not be reported. It is the opinion of the writer that the studies included in the research review were typical of most of those available.

One major point of agreement, in most studies, was that the instructor of the large class was instrumental in determining the success of the class. There is also agreement that much more research needs to be done regarding large-group instruction.

In contrasting or comparing large to regular size classes, there are some major areas of disagreement. Instructors, favoring regular size classes believe that large classes are not as effective as regular size classes with regard to the following:

1. Stimulating pupil interest
2. Providing for individual differences
3. Developing favorable attitudes
4. Achievement and understanding acquired by the students.

However there is insufficient research evidence to verify these conclusions.

CHAPTER III

THE DEVELOPMENT OF A COURSE FOR LARGE-GROUP INSTRUCTION

Initial Developments of the Course

The College of Education, the Florida Institute for Continuing University Studies, and representatives of Duval County were primarily involved in the planning for a course in modern mathematics for a large group of elementary school teachers. The College of Education would provide the principal instructors for preparing the course outline and conducting the class. The course was being designed for off-campus; therefore it would be offered through the Florida Institute for Continuing University Studies. The course would be taught in Duval County; hency the County arranged for the necessary facilities. The title of the course was MS 300: Theory of Arithmetic.

During the fall trimester of 1963, Leroy Smith, Mathematics Supervisor of Duval County, met with the writer to discuss an in-service training program in mathematics for Duval County teachers. One of the most pressing problems was to offer courses in modern mathematics for the elementary school teachers. The County has over two thousand teachers in its elementary schools. If a course were offered each trimester with enrollment limited to thirty-five teachers, it would be impossible to meet the county's needs. It would require more than ten years to reach all of these teachers.

The idea of offering a course for a large group of elementary school teachers was discussed. It was decided that this was a definite possibility and would be pursued. Leroy Smith agreed to work out any administrative problems with the county, arrange facilities, and inform all of the elementary school principals. The principals were to notify the teachers. The writer agreed to submit the proposal to the College of Education and the Florida Institute for Continuing University Studies.

The writer met with Dr. Kenneth P. Kidd of the College of Education and Dr. Kidd worked out the necessary details with the University of Florida to offer the course. Dr. Kidd and the writer arranged a meeting with Dr. Sam Moorer, Associate Dean of the Florida Institute for Continuing University Studies. A decision was made in favor of conducting the course. The course was offered in the spring trimester of 1964. The class enrollment was limited to two hundred and twenty-five elementary school teachers.

Planning the Course

Certain key questions were raised in planning for the course:

(a) Can the effectiveness, usually associated with regular size classes, be maintained in a class with such a large enrollment?

(b) How much of an instructor's teaching load should be allotted for conducting the large class?

(c) How can class time be best utilized to care for special interest groups?

(d) What evaluation procedures should be used to determine the effectiveness of the class?

(e) What teaching procedures are best suited for this type of instruction?

Off-campus classes with a normal enrollment usually have fourteen three-hour instructional meetings. However, it was decided this large class would have sixteen three-hour sessions. This would allow for organizational and evaluation meetings. The writer was given the primary responsibility for planning and conducting the class. Two-thirds of a teaching load would be allotted for this course. Mr. Richard Grubaugh, University of Florida, would be allotted one-third time to work with the writer.

A decision was made to divide each three-hour session into two equal periods. The first one and one-half hour session would be devoted to instruction of the entire group through lecture/demonstration. This session would be conducted by the writer. The large class would be divided into subgroups for the second one and one-half hour sessions. The students (elementary school teachers) would be randomly assigned to subgroups of not more than thirty students. E ch subgroup would be assigned an instructor to conduct this one and one-half hour session. The subgroup instructors would be rotated to a new class approximately every other meeting. This was an effort to remove the influence of a given instructor and also to allow each subgroup the benefit of the better instructors.

The instructors for the subgroup sessions were selected from Duval County. The selections were based on excellent recommendations of Leroy Smith, Mathematics Supervisor; recency of work in modern mathematics; recommendations of principals; expressed interest in working with the elementary school teachers; and knowledge the writer possessed regarding the applicants for subgroup instructors. The following mathematics teachers were finally selected to serve as subgroup instructors: Daryle C. May, Xylda C. Stanley, Joseph R. McDaniel, Evelyn Underwood, Myrtle Williams, Rene Owen, Robert Davie, and Ralph Balyeat. The Florida Institute for Continuing University Studies approved these Duval County mathematics teachers and agreed to pay each $15 per class meeting.

Subgroup Instructors' Duties

The instructor of each subgroup was to divide the one and one-half hour session as follows: approximately thirty minutes would be devoted to discussion of ideas introduced in the large class session and questions related to previous homework assignments; the remainder of the session would be devoted to working assignments related to the topics introduced in the large class session and to examining modern elementary school mathematics programs. A library of modern materials would be made available to each subgroup. These materials could be checked out each meeting. During the latter part of the subgroup session, each subgroup instructor would devote his time to giving individual help to the students.

The subgroup instructors would also be responsible for taking roll and administering tests. These instructors would meet forty-five minutes before each three-hour session begins. They would be given graded assignments and materials, receive assignments to hand out in the subgroups, and be briefed on major ideas to be introduced in the large-group session. They would be required to attend the large-group sessions. A thirty-minute session would be held, after the three-hour session, with the subgroup instructors. All assignments from the subgroups were to be turned in to the writer and any questions from the subgroup instructors would be answered. They would receive the assignments the students were to receive the next meeting. In this way the subgroup instructors could have adequate time to make preparation.

A student assistant at the University of Florida would be paid by the Florida Institute for Continuing University Studies to grade papers and type all assignments and materials.

Special Interest Groups

To meet the needs of special interest groups more fully, two additional provisions are planned. First, teachers who were especially interested in a particular grade level, or a more extensive development of a given topic (sets, fractions, etc.) would form a special subgroup to be conducted each meeting. The first forty-five minutes of the second one and one-half hour session would be devoted to this purpose. This meant that students especially interested in the special subgroup topic would miss the first forty-five minutes of their regular subgroup session. Each week teachers of a particular grade level would be invited to the special subgroups. Topics from the modern elementary mathematics programs at that particular grade level would be introduced or more extensively developed. This implies that different students would be attending the special subgroup meetings each session. These special subgroup sessions were to be conducted by Mr. Richard Grubaugh or the writer. At the end of this special session, the students would return to their regularly assigned subgroup. This provision allowed time for these students to share in their regular subgroup assignments and receive individual help from the subgroup instructor.

Secondly, the elementary school teachers enrolled in the large class would be given the opportunity to invite the writer to their schools to teach demonstration classes or talk to school faculties. One day each week would be devoted to this purpose. This provision is a direct effort to stimulate faculty interest in trying out new ideas in the modern elementary mathematics programs. It should also serve to establish good working relations between the elementary school teachers enrolled in the course and the writer.

Large-Group Session

The large-group session would be conducted primarily with the use of a large overhead projector and screen. Many overlays of examples and illustrations would be worked out on various topics. This session would be used primarily to introduce topics and stimulate interest.

Supplementary materials and assignments would be worked out for each large-group lecture where note-taking could be minimized. The students would be allowed to ask questions during this lecture period.

Mr. Richard Grubaugh would assist the writer in the preparation of materials and assignments for the large-class session. It was decided to work out a complete outline for the large-class session so materials could be prepared well in advance.

Development of the Course Outline

Purpose and Aims of the Course

The primary goal of the course was giving the elementary school teachers an opportunity to become better acquainted with the structure of arithmetic. Major emphasis was placed on understanding the operations and properties of the real number system. The elementary school teacher would be given assignments directly related to topics in modern elementary mathematics programs. Topics and approaches in these programs would be stressed. Hence, the major purposes were directed toward improving instruction in arithmetic by affording the elementary school teachers an opportunity to gain more understanding in mathematics.

The elementary school teachers enrolled in the course would be encouraged to experiment in the classroom with "new ideas" introduced that were appropriate for their respective grade levels. In many cases, the writer would visit classrooms and teach demonstration classes at the request of the teachers.

Content Outline for the Course

The major lecture headings were:

1. Set Language and Notation
2. The Set of Whole Numbers - Operations and Properties
3. The Structure of Whole Numbers - Factors, Multiples, and Primes
4. Elementary Number Theory
5. Bases Other Than Ten - A Grouping Scheme
6. The Set of Integers - Operations and Properties
7. The Set of Positive Rationals - Operations and Properties of Common Fractions
8. Decimals and Percentage
9. The Set of Irrationals - Extracting Roots, Properties and Operations

10. The Structure of the Real Number System
11. Measurement
12. Geometry in the Elementary Schools
13. Problem Solving
14. Writing Number Sentence - Equations and Inequalities
15. Other Mathematical Systems

Assignments and materials were worked out in advance on these topics. A test was selected and partial weekly assignments were given from this text. The text selected was Fundamentals of Arithmetic for Teachers, by Clifford Bell, Clela Hammond, and Robert Herrera, published by John Wiley and Sons, Inc., New York, 1962.

Evaluation Procedures To Be Used

A test for measuring understandings of arithmetic concepts would be given at the first and last meetings of the course. This particular test would be given for diagnostic and comparative purposes. Other tests would be used for grading purposes throughout the trimester. An assignment would be given covering each topic introduced and these assignments also would be used for grading.

Opinionnaires were to be filled out by the students. These opinionnaires would be used by the writer to pace the introduction of topics and to gather data concerning the students' attitudes toward the course. A description of the instruments and how they were used will be described in the following chapter.

Summary

The course, MS 300, The Theory of Arithmetic, as offered for large-group instruction was a cooperative venture involving the College of Education, The Florida Institute for Continuing University Studies, and Duval County. The course was offered during the spring trimester of 1964 and met sixteen three-hour sessions. Meetings were scheduled each week. Facilities were arranged by Duval County and the course was conducted by the writer, assisted by Mr. Richard Grubaugh, and eight mathematics teachers from Duval County.

The course was conducted in accord with the plans described in this chapter. In addition to regular class meetings, a total of sixty-one demonstration classes, including faculty lectures, was taught by the writer in the Duval County schools. These demonstration classes were considered highly valuable by principals and teachers of the schools involved. It was considered a very desirable feature of the course by the writer. However, no attempt was actually made to measure its effectiveness.

A total of two hundred and twenty-one students enrolled in the course. This enrollment included those teachers enrolled for three semester hours college credit as well as non-credit.

CHAPTER IV

THE EVALUATION INSTRUMENTS AND COMPARISON PROCEDURES

The Evaluation Instruments

Purpose of the Instruments

Three basic forms of evaluation instruments were used in this study: an instrument designed to measure mathematical understandings, an attitude scale to diagnostically determine attitudes toward mathematics, and a course evaluation form which allowed students to express their opinions as to the relative effectiveness of the course. These instruments served to gather all data as to the relative effectiveness of the large-group instructional plan. Other class tests were given but these tests were used only for grading purposes. The primary evaluation instrument was the test devised to test for basic mathematical understandings.

A Test of Basic Mathematical Understandings

A number of tests devised to measure growth of mathematical understanding were examined by the writer. One such test had been constructed by Dr. Kenneth P. Kidd, University of Florida. The writer requested Dr. Kidd's permission to use this test in the study.

The test contained forty items, multiple choice, with four choices for each item. The test had been subjected to item analysis procedures on two different occasions. Dr. Kidd and the writer agreed the test should be expanded for a broader coverage. With Dr. Kidd's assistance, the writer expanded the test to seventy-eight items. A decision was made to maintain four choices for each item. This test was then administered to one hundred and ten in-service and prospective elementary school teachers. These teachers represented classes taught by Dr. Kidd and Dr. Waldemar Olson. An item analysis was used to study the top and bottom 27 per cent of the scores. The item analysis procedure used was suggested by N. M. Downie (4). The item analysis included a study of the distractors for each item. From the seventy-eight item test, a total of sixty-seven items was selected. Some of these items were rewritten, including distractors. This sixty-seven item test was used as a pretest and a posttest in the study. A copy of the test appears in the appendix.

The test was administered as a pretest and, using these papers, a second item analysis was made. The item analysis data are shown below in Table 1. The difficulty is an estimate which is found by averaging columns (1) and (2).

TABLE I

ITEM ANALYSIS
Test of Mathematical Understanding

	(1)	(2)	(3)	(4)
	Per Cent Correct			
Item	Upper 27%	Lower 27%	Difficulty	Discrimination
1	72	35	54	38
2	98	78	88	50
3	89	45	67	50
4	24	15	19	12
5	46	13	30	40
6	93	52	72	55

	(1)	(2)	(3)	(4)
	Per Cent Correct			
Item	Upper 27%	Lower 27%	Difficulty	Discrimination
7	50	13	32	43
8	56	35	45	21
9	82	17	50	63
10	87	39	63	51
11	96	76	86	40
12	82	33	57	48
13	48	20	34	32
14	26	15	22	17
15	50	13	31	43
16	48	07	28	59
17	63	26	44	38
18	74	30	52	45
19	78	35	57	44
20	91	54	72	45
21	95	50	73	60
22	91	46	68	52
23	48	19	34	32
24	89	46	67	50
25	69	20	45	50
26	69	35	52	36
27	39	13	26	35
28	74	54	64	22
29	59	13	36	50
30	48	33	40	19

	(1)	(2)	(3)	(4)
	Per Cent Correct			
Item	Upper 27%	Lower 27%	Difficulty	Discrimination
31	67	32	50	35
32	31	13	22	25
33	80	41	60	41
34	91	35	63	60
35	96	24	60	75
36	96	48	72	61
37	43	07	25	50
38	63	02	33	80
39	91	63	77	40
40	74	22	48	53
41	91	24	57	68
42	85	15	50	68
43	85	19	52	65
44	87	30	59	58
45	85	17	51	67
46	72	32	52	41
47	57	07	32	60
48	78	20	49	58
49	37	04	21	50
50	45	17	31	35
51	71	13	42	60
52	72	37	54	37
53	69	30	50	40
54	89	22	55	68

Item	(1) Per Cent Correct Upper 27%	(2) Lower 27%	(3) Difficulty	(4) Discrimination
55	82	09	46	72
56	98	52	75	65
57	91	43	67	55
58	56	06	31	61
59	82	28	55	53
60	63	17	40	48
61	78	19	48	59
62	91	22	57	70
63	71	26	49	45
64	37	19	28	21
65	89	32	61	60
66	72	26	49	47
67	46	22	34	28

Downie (4) states that E. B. Davis has shown that maximum discrimination of an achievement test which would discriminate throughout the range is obtained when the distribution of item difficulties or average item difficulty tends to be around 50 per cent. Items selected for an achievement test should range from very easy to very difficult, with the majority clustering near the middle around the 50 per cent level.

A nomograph was used to compute the index of discrimination or actually the correlation coefficient. The nomograph used was a modification of a nomograph developed by J. C. Flanayan (8). Every item in the above table shows positive discrimination. Downie (4) points out that any item which has an index of .20 or above is actually working. This is considering a hundred or more papers. This item analysis involved one hundred and eight scores.

The Attitude Scale

The attitude scale used in the study was developed by Wilbur H. Dutton (5) (6) of the University of California, Los Angeles. The details of the test construction appears in the Elementary School Journal (September, 1954, pp. 24-31). The writer examined the scale, requested and received permission from Wilbur H. Dutton to use it in this study. A copy of this scale appears in the Appendix. The weight, or scale value, of each item is inserted. Students having average scale value scores between 1.0 and 5.5 are considered to have unfavorable attitudes toward arithmetic.

The attitude scale was used as a pretest and posttest instrument. This was an effort to detect any comparative changes of attitudes.

Course Evaluation Form

The course evaluation form was constructed by the writer, with assistance from Dr. Kidd. This form was administered at the twelfth class meeting. The writer selected this time so that the data might be used to improve the instruction plan. This form gave the class an opportunity to express their opinions on many aspects of the course. The results appear in the following chapter and a copy of this evaluation form is in the Appendix.

Regular Classes for Comparison

Three regular-size classes were selected for comparative purposes. Two of the classes were taught by Richard Grubaugh, University of Florida, and one course by Dr. Waldemar Olson, University of Florida. The two classes taught by Mr. Grubaugh were off-campus classes. One class was located in Cocoa and one in Yulee. Both courses were MS 300. Mr. Grubaugh also assisted the writer in the large-group instructional plan. The same instruments for gathering data in the large classes were used in the small classes, excluding the evaluation form. Mr. Grubaugh followed the same basic development of topics.

The regular-size class conducted by Dr. Olson was an on-campus class. This was a course in methods of teaching arithmetic. Basically the same topics as covered by the test were covered by Dr. Olson. The same pretest and posttest instruments were administered by Dr. Olson.

These data from regular size classes were used to compare the large group instructional plan. However, no classroom visitations--demonstration teaching--were made for the teachers in either Mr. Grubaugh's or Dr. Olson's classes.

Summary

To evaluate the effectiveness of the large-group instructional plan, three basic evaluation instruments were used. These instruments consisted of a test of mathematical understanding, an attitude scale, and a course evaluation form. The test of mathematical understanding and the attitude scale were used as a pretest and posttest in the large class as well as the regular size control classes.

The regular size control classes, which were used for comparative purposes, were two off-campus classes and one on-campus class. The two off-campus classes were taught by Mr. Richard Grubaugh and the on-campus class was taught by Dr. Waldemar Olson.

The large-group instructional plan allowed time for the writer to visit the schools of the elementary school teachers enrolled in the large class, and teach demonstration classes. This was not a part of the regular size classes.

CHAPTER V

THE FINDINGS OF THE STUDY

The presentation of the findings of the study will consist of three sections: first, findings regarding mathematical understandings; second, findings regarding attitudes; and finally, the students' evaluation of the large class. The pretest and posttest raw data regarding mathematical understandings and attitudes appear in the appendix. The data are ranked according to the pretest scores on mathematical understandings.

There were two hundred and three students in the large class. One hundred eighty-six students took both the pretest and posttest on mathematical understandings and attitudes. The seventeen students not included were mainly non-credit and late registrants. This was the first of such classes to be taught in Duval County. One hundred eighty-four of the one hundred eighty-six students included in the large class were white teachers.

The regular size classes in Gainesville, Cocoa, and Yulee had twenty-four, thirty-six, and eighteen students respectively. These classes also consisted of predominantly white teachers. Furthermore all classes involved in this study consisted of teachers who enrolled voluntarily.

Mathematical Understanding

Figure I shows the means and ranges of scores on the test of mathematical understandings. By means of this figure, the reader may

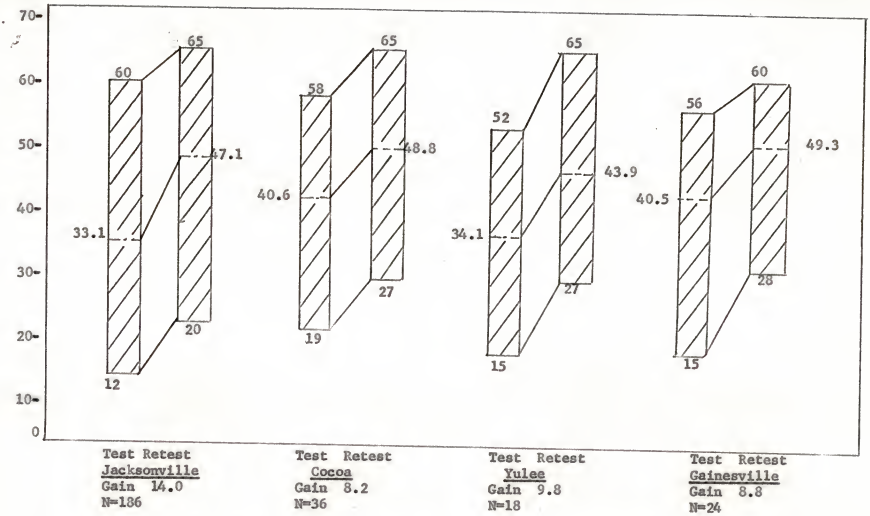

Figure I. Means and Ranges of Scores on Pretest and Posttest of Mathematical Understanding

obtain an overall view of the growth that took place in each of these classes.

The measured gains, as shown in Figure I, for the three regular size classes are approximately the same. The large class gain exceeds the mean gain of any one of the regular size classes. However, if the experiment were retried, it is highly probable that changes in practical factors would have some effect on the results. For example, a class with a forced enrollment may show less favorable gains in mathematical understandings than a class in which the enrollment is voluntary. It is possible that a class with forced enrollment would be faced with a morale problem. If the overall class morale was low, the development of a good learning situation would be hindered. The initial mathematical competency level should also be considered by a researcher. A class with an initially low mathematical competency level may show less favorable gains than a class with an initially high mathematical competency level. This study did not attempt to evaluate the extent that the results of instruction were affected by the personality of the instructor and his approaches to teaching. With another instructor using a different approach to teaching similar results may not be reproduced.

Generally speaking, Figure I shows that the large class compared favorably with the three regular size classes, especially since the large class does have the higher mean gain.

Table 2 shows other data relative to the large and regular size classes. The means and ranges are included in these data. These data do not reveal any large differences between the pretest and posttest

TABLE 2

PRETEST AND POSTTEST DATA ON MATHEMATICAL UNDERSTANDINGS

Item	Class Location			
	Jax N=186	G'ville N=24	Cocoa N=36	Yulee N=18
Pretest mean	33.13	40.54	40.58	34.06
Posttest mean	47.09	49.38	48.81	43.89
Gain	13.96	8.84	8.23	9.83
Pretest maximum score	60	56	58	52
Pretest minimum score	12	15	19	15
Range	48	41	39	37
Posttest maximum score	65	60	65	65
Posttest minimum score	20	28	27	27
Range	45	32	38	38
Pretest variance	100.89	94.61	127.39	111.64
Posttest variance	90.52	70.59	90.28	86.69
Pretest standard variation	10.04	9.73	11.29	10.55
Posttest standard deviation	9.51	8.40	9.50	9.31

variances of the regular size classes and the large class. In each case the large class does make a favorable comparison.

In an effort to make an internal comparison of the data from the large class, the members were put into three equal groups on the basis of their pretest scores on mathematical understanding. Table 3 shows the actual mean gains and the possible mean gains for the top, middle, and bottom one-third of the class.

TABLE 3

ACTUAL AND POSSIBLE MEAN GAINS OF THE TOP, MIDDLE, AND BOTTOM ONE-THIRD OF THE LARGE JACKSONVILLE CLASS ON MATHEMATICAL UNDERSTANDINGS
(N=62 for each third)

Item	Actual Mean Gain	Possible Mean Gain	Gain as a Per-cent of Pos-sible Gain
Top one-third	9.50	22.74	42
Middle one-third	14.29	33.59	43
Bottom one-third	18.23	45.09	40

If the actual mean gain of each group is examined, it appears that the bottom and middle sections accounted for the greater part of the total class gain. However, one must consider the fact that if a student scored high on the pretest, he could show only a small actual gain. Hence if a student scores sixty on the pretest, he can show a maximum gain of only seven. This is assuming a perfect score on the posttest. This is not to say that this seven point gain represents all the student can or has learned in the class. This test, or any test, can actually measure everything a student learns in a given course. Hence the limitations on the test must be given considerations. However, in terms of the test, a student scoring low on the pretest can possibly show a large gain. Table 3 shows not only the actual mean gains but also the possible mean gain and the gain as a percent of possible gain. The possible mean gain for the top one-third of the class is obviously lower than the possible gain of the middle or the possible gain of the bottom one-third. The possible mean gain of the bottom one-third of the class is the largest of the three groups. In terms of gain as a percent of possible gain, Table 3 shows that each group did equally as well in terms of possible growth. Hence, the bottom and the middle one-third of the class did no better than the top one-third in terms of possible growth. The students in the three small comparison classes were pooled and also placed into three groups on the basis of the scores on the pretest on mathematical understandings. Table 4 shows the actual mean gains and the possible mean gains for the top, middle and bottom one-third of the pooled class.

In the pooled regular size classes, Table 4 shows that the top one-third of the class has the highest percent of possible gain. However, as with the large class, in terms of only the actual mean gain, it appears that the bottom and middle one-third of the pooled class accounted for the greater part of the total class gain.

If each respective group of the pooled regular size classes is compared with the large class, the data show the large class does make a favorable comparison. In each case the large class shows a higher mathematical gain.

TABLE 4

ACTUAL AND POSSIBLE MEAN GAINS OF THE TOP, MIDDLE, AND BOTTOM ONE-THIRD OF THE POOLED REGULAR SIZE CLASSES ON MATHEMATICAL UNDERSTANDINGS
(N=26 for each third)

Item	Actual Mean Gain	Possible Mean Gain	Gain as a Percent of Possible Gain
Top one-third	5.69	15.35	37
Middle one-third	9.04	27.88	32
Bottom one-third	12.31	40.46	30

Attitudes

An attitude scale developed by Wilbur Dutton (5) (6) was used to collect all data relative to attitudes. A description of the scale was given in Chapter IV. The scale was used as a pretest and a posttest instrument in all classes involved in this study. Table 5 shows the means and gains on attitudes for each class involved in the study. Each class had a slight mean gain in attitudes. However only slight difference existed in the pretest mean and the posttest mean for each class. This similarity of attitude scores is also reflected in the range, variances, and standard deviations for each of the classes. Excluding Yulee less than one point separates the pretest attitude means of these classes. However less than one point separates all four classes on the posttest attitude means. In each case, the large class makes a favorable comparison relative to the regular size classes.

TABLE 5

PRETEST AND POSTTEST DATA ON ATTITUDES TOWARD ARITHMETIC

Item	Class Location			
	Jax	G'ville	Cocoa	Yulee
Pretest mean	6.4	6.7	6.9	5.6
Posttest mean	7.0	7.4	7.1	6.5
Gain	0.6	0.7	0.2	0.9
Pretest maximum score	9.3	8.7	9.1	8.1
Pretest minimum score	2.1	3.4	2.0	2.8
Range	7.2	5.3	7.1	5.3
Posttest maximum score	8.9	9.0	9.1	8.9
Posttest minimum score	2.1	3.9	2.2	3.4
Range	6.8	5.1	6.9	5.5
Pretest variance	3.54	2.52	4.02	3.53
Posttest variance	1.50	1.79	2.12	3.01
Pretest standard deviation	1.89	1.59	2.01	1.88
Posttest standard deviation	1.22	1.41	1.45	1.74

Initially in the large class 28 percent of the students scored below 5.5 on the pretest attitude scale and only 15 percent scored below 5.5 on the posttest attitude scale. However, it should be noted that the students who scored below 5.5 on the posttest did not in all cases score below that point on the pretest.

A comparison of the percent of students scoring below 5.5 on the pretest and posttest can be made from Table 6. Students scoring below 5.5 are considered to have unfavorable attitudes toward arithmetic.

TABLE 6

THE PERCENT OF STUDENTS SCORING BELOW 5.5 ON THE PRETEST ATTITUDE SCALE AND THE PERCENT SCORING BELOW 5.5 ON THE POSTTEST

Class	Percent of Students Below 5.5		Net Change
	Pretest	Posttest	
Jacksonville (Large Class)	28	15	13
Gainesville	25	8	17
Cocoa	19	19	0
Yulee	39	28	11

Table 6 shows that the regular size class in Gainesville has the highest net change. The large class showed the second highest net change. Table 6 shows an overall trend toward more favorable attitudes. Excluding Cocoa, each class had a smaller percent of students below 5.5 on the attitude posttest than on the initial attitude pretest.

A bivariate frequency distribution, sometimes called a scatter diagram or a correlation table, was made up for each class using pretest mathematics scores and pretest attitude scores. Figures II, III, IV, and V show the relation between pretest mathematics scores and pretest attitude scores for each class involved in the study. The correlation coefficient is also given with each corresponding figure. The Gainesville regular size class shows numerically the least correlation. Actually, other than Yulee, none of these classes show a very high correlation between the respective pairs of scores.

One might suppose that there is a tendency for teachers who have a high level of competency in arithmetic also to have the most favorable attitudes toward arithmetic and teachers with a low level of arithmetic competency to have the most unfavorable attitudes. However, excluding Yulee, these figures show very little correlation between pretest mathematics scores and preattitude scores. Actually there is no consistency shown on the scatter diagrams. Hence, a low score on the mathematics pretest will not necessarily correspond to a low score on the attitude pretest. The statistics imply that teachers with a low competency level in mathematics may have a highly favorable attitude toward mathematics--an implication somewhat contrary to what is the general belief. Of all the classes, the one in Yulee shows the highest correlation between pretest attitudes and pretest mathematics scores. Yulee also had the greatest number of students to score below 5.5 on the pretest attitude scale.

Kelley (*) suggests a method of comparing correlation coefficients obtained from two sets of data having different ranges.

> In the case two different series of measures are correlated it is usually not known just what is the nature of the curtailment or extension of the ranges of the two series which has been brought about by some selective agency. . . . If,

	Pretest Attitude Scores							
Pretest Mathematics Scores	2.0-2.9	3.0-3.9	4.0-4.9	5.0-5.9	6.0-6.9	7.0-7.9	8.0-8.9	F
55-59						1	1	2
50-54			1	1		1		3
45-49						3	1	4
40-44			2			3		5
35-39		1		1	1			3
30-34				1		2	2	5
25-29					1			1
20-24								
15-19					1			1
10-14								
F		1	3	3	3	10	4	24

Figure II. Pretest mathematics scores shown in relation to pretest attitude scores for the Gainesville regular size class (r = .06)

	Pretest Attitude Scores							
Pretest Mathematics Scores	2.0-2.9	3.0-3.9	4.0-4.9	5.0-5.9	6.0-6.9	7.0-7.9	8.0-8.9	F
55-59								
50-54						1		1
45-49						2		2
40-44		1			1		1	3
35-39			1					1
30-34		1		1	2	1		5
25-29				2	1			3
20-24		1						1
15-19	1	1						2
10-14								
F	1	4	1	3	4	4	1	18

Figure III. Pretest mathematics scores shown in relation to pretest attitude scores for the Yulee class (r = .68)

Pretest Mathematics Scores	Pretest Attitude Scores							
	2.0-2.9	3.0-3.9	4.0-4.9	5.0-5.9	6.0-6.9	7.0-7.9	8.0-8.9	F
55-59							1	1
50-54				1	2	4	2	9
45-49	1	1	1	1	1	10	4	19
40-44		1		1	4	9	5	20
35-39	6	1	3	2	4	15	5	36
30-34	1		3	3	9	12	5	33
25-29	4	3	3		3	7	4	24
20-24	2	2	2	5	3	6	3	23
15-19	1	3	2	4	3	3	2	18
10-14	1		1			1		3
F	16	11	15	17	29	67	31	186

Figure IV. Pretest mathematics scores shown in relation to pretest attitude scores for the large Jacksonville class (r=.29)

Pretest Mathematics Scores	Pretest Attitude Scores							
	2.0-2.9	3.0-3.9	4.0-4.9	5.0-5.9	6.0-6.9	7.0-7.9	8.0-8.9	F
55-59							3	3
50-54		1			1	2	2	6
45-49						3	3	6
40-44					3	2	2	7
35-39			1		1	1		3
30-34	1					1	1	3
25-29	1	1			2			4
20-24			1				2	3
15-19	1							1
10-14								
F	3	2	2		7	9	13	36

Figure V. Pretest mathematics scores shown in relation to pretest attitude scores for the Cocoa class (r=.52)

> however, the nature of the curtailment is known and is upon the basis of one trait only we may derive a formula enabling a comparison of correlation coefficients obtained from different ranges. Note that one trait is arbitrarily curtailed (or extended) and that the other is affected only in a consequential manner. (9:224)

In an effort to find an explanation for the lack of consistency between the measures of correlation, the writer used Kelley's technique to adjust the ranges of the data in the other groups to that of Yulee. However, the differences in ranges was found to be non-contributing to the differences in the correlation measures. For example, the Gainesville correlation was raised from .06 to only .07. Hence no basic correlation was shown to persist between pretest mathematics scores and pretest attitude scores. Again, a teacher with a low level of competency in mathematics may have a favorable attitude toward mathematics.

Bivariate charts for each class were also constructed to show the relationship between gains in mathematics and gains in attitudes. However, the charts show little or no relationship between the two variables. From these charts, students making large gains in mathematics do not necessarily make large gains in attitude. A bivariate chart was also constructed by selecting the mathematics and attitude scores of large class students scoring below 5.5 on the pretest attitude scale. Gains in mathematics were shown in relation to gains in attitudes. The bivariate chart showed little or no relationship existing between the two variables. All bivariate charts showing gains in mathematics in relation to gains in attitudes appear in the appendix.

The Students' Evaluation of the Large Class

An evaluation form was constructed for the large class members. The form was used so that students could express their opinions as to the effectiveness of the large class. Table 7 shows the responses of the students in the large class. The number of students responding to each item is inserted. The students were asked to state two features they particularly liked and also two features they did not like about the course. Table 8 and Table 9 show a summary of the responses.

Most of the favorable comments made by the class members were related to the idea that they were receiving greater understanding of mathematics. This observation is especially interesting since the main objective of the course was to emphasize the structure or understandings of mathematics. For example, comments regarding illustrations and examples, interesting approaches to topics, an emphasis on "why" rather than just memorization, and different approaches to solving problems are all related to "teaching for understanding."

The length of the homework assignments and the amount of content covered received the majority of the unfavorable comments. If the fact is considered that the teachers enrolled in the class have completed a full day's work by the time they come to class, it is understandable that homework assignments would receive unfavorable comments.

As far as an overall evaluation of the course was concerned, the vast majority of the class members were highly complimentary of both the lectures for the large group and the discussions carried on in the small groups. Furthermore, to the question "If a sequence to this course were

TABLE 7

EVALUATION OF COURSE BY STUDENTS IN LARGE CLASS

---

1. How significant do you consider this course?
   No Value: 0. Some Value: 34. Extremely Valuable: 157.

2. Have "new ideas" been presented that were useful in your classroom?
   None: 2. Some: 73. Many: 116.

3. To what extent has your interest been aroused?
   Little or no interest: 0. Some interest: 49. Extremely interested: 142.

4. Have the small class discussions been helpful?
   No: 7. Somewhat helpful: 62. Very helpful: 121.

5. Has the pace of the instructor (large class) been satisfactory?
   Too slow: 12. Just right: 128. Little too fast: 43. Too fast: 6.

6. To what extent has the content and ideas been "new" to you?
   None new: 0. Some new: 111. Mostly new: 80.

7. Have you felt free to ask questions in the small classes?
   No hesitancy: 164. Some hesitancy: 22. Great hesitancy: 2.

8. Do you have difficulty in knowing what to read or study or what is expected of you?
   No difficulty: 135. Some difficulty: 53. Lot of difficulty: 3.

9. How meaningful have the illustrations and explanations been to you?
   No value: 1. Some were meaningful: 51. Very meaningful: 139.

10. Would you consider it worthwhile for other teachers to take this course?
    Not worthwhile: 0. Worthwhile to a few: 17. Extremely worthwhile: 173.

11. If a sequel to this course (MS 305) were offered at a later date, would you be interested in taking that course?
    No: 32. Possibly: 74. Yes: 87.

12. How well have you understood the content presented?
    None of it: 0. Some of content: 33. Most of content: 155.

TABLE 8

OPINIONS CONCERNING THE BEST FEATURES OF COURSE

79	The lecture by the instructor emphasizing essential concepts.
63	I felt that the small discussion groups were very effective.
49	This course was valuable because it taught the "why" of arithmetic, not just how to memorize facts and figures.
32	The examples used in class were helpful in clarifying the subject matter.
29	This course aroused new interest, confidence, and enthusiasm for mathematics.
27	The assignments were well thought-out and helped me understand the course.
19	The explanations of homework were adequate
18	The use of the overhead projector contributed to the comfort and attention of the whole class.
16	I enjoyed being exposed to the new ideas.
15	The new material was applicable in the elementary classroom.
13	The entire course was well planned.
11	There was adequate class time to work on assignments.
10	The new methods demonstrated in this course make mathematics more interesting to children.
9	Finding a different approach to solving problems was an interesting and valuable part of the course.
6	This course was an inspiration to me to do a better job in my classroom.

TABLE 9

OPINIONS CONCERNING THE POOREST FEATURES OF COURSE

35	The assignments were a little too long.
32	There was not enough time to grasp all of the content presented in this course.
19	The lecture group was too large.
15	Poor study conditions and noise in the large and small groups made it difficult to concentrate.
14	I was too tired after teaching all day really to get the most from this course
14	The small discussion groups were too long.
9	The class met at an inconvenient time.
8	There should be more emphasis on how to teach.
8	There should have been more classroom application of what we learned in this course.
7	The discussions in some of the small groups were monopolized by a few of the students.
6	The small groups should be arranged by grade levels.
5	This course should also be offered in the fall, and/or the summer.
5	The small group instructors didn't understand the content of the course well enough to explain it clearly.
5	The class break was too long.

offered, would you be interested in taking that course?" Eighty-six percent of the respondents indicated that they were either "definitely interested" or "probably interested."

Summary

The evaluation of the findings have been presented in three parts: mathematical understandings; attitudes; and a course evaluation by the students.

The large class showed as much gain in mathematical understandings as did any one of the regular size classes. Actually, the large class showed a slightly higher mean gain. Each class involved in the study showed only a slight gain in attitudes. A comparison of preattitude test means and posttest attitude means showed only slight differences for each class involved.

These data did not support the hypothesis that there existed a high correlation between the attitudes held by elementary school teachers toward mathematics and their level of understanding of mathematical concepts. Furthermore little or no relationship was shown to exist between gains in understanding of mathematical concepts and gains in attitudes toward mathematics.

The favorable evaluation given by the students of the large class indicate that its organization and instruction were satisfactory.

CHAPTER VI

SUMMARY, CONCLUSIONS, AND IMPLICATIONS

Summary

The basic purpose of the study was to investigate the effectiveness of large-group instruction. Specifically the study was to seek answers to the following questions: Will the effectiveness of large-group instruction, followed by small discussion groups and demonstration teaching, be comparable to that of classes of regular size? More particularly, do teachers learn well under the conditions of this experiment? Do they acquire the essential psychological understandings?

The effectiveness of large-group instruction was considered in terms of mathematical understandings, students' attitudes toward mathematics, and the students' evaluation of the course.

This study described the need for large-group instruction and the planning involved in conducting the experiment. The University of Florida, Florida Institute of Continuing University Studies, and Duval County were the primary agencies involved in initiating and offering the class. It was the first class of this nature to be offered in Duval County. The course was taught by the writer.

The class organization was based on the assumption that class instruction in mathematics for elementary school teachers involves two types of activities which can be separated: first, a lecture in which the main mathematical concepts are presented to the total group; second, sessions in which class members are encouraged to raise questions and discuss solutions to these questions. Each of the sixteen three-hour weekly meetings of the large class was divided into a one and one-half hour discussion period. Discussion groups consisted of twenty to twenty-five randomly selected class members. Each of these groups was under the direction of a carefully selected group leader who, in turn, was briefed and supervised by the instructor. Special provisions were also made for special interest groups. These provisions were described in Chapter III.

In addition to lecturing and supervising the leaders of the small discussion groups, the writer visited schools and taught demonstration classes. A total of sixty-one demonstration classes was taught. These classes were given upon invitation from elementary school teachers who were enrolled in the course. This feature was considered highly desirable by the teachers and by the principals of their respective schools. This feature was not a part of the regular size classes.

Three basic evaluation instruments were used in this experiment: first, the primary instrument was a test of mathematical understandings which was used as a pretest and posttest; second, an attitude scale which was used as a pretest and posttest instrument; and finally, a course evaluation form which was to be filled in by the students.

Three regular size classes were used for comparison purposes. The attitude scale and the test of mathematical understandings were also administered to these classes.

Conclusion

The findings of this experiment suggest certain conclusions; however these conclusions should be regarded in the light of certain delimitations. As previously stated, a change in practical factors may have effects on the results of the experiment. For this experiment, all classes consisted of teachers who enrolled voluntarily. It was the first such course offered in Duval County. One hundred eighty-four of the one hundred eighty-six students included in the large class experiment were white teachers. The course was for teachers of elementary school mathematics; hence, it was restricted to one subject area. As previously stated, no attempt was made to evaluate the extent that the results of instruction were affected by the personality of the instructor and his teaching procedures. With another instructor using a different approach to teaching, similar results may not be produced.

Keeping in mind the delimitations previously stated, the conclusions of this experiment are:

1. Large-group instruction may be effectively utilized for off-campus in-service education of elementary school mathematics teachers.

2. The students of the large class compared favorably, in terms of mathematical gains and favorable changes in attitudes, with students in each of the regular size classes.

3. No substantial correlation was shown to exist between mathematical competency and attitudes toward mathematics.

Implications

Large classes should be seriously considered as an effective means for the training of elementary school teachers in mathematics. There are probably a variety of organizational plans for such large-group instruction. The organizational plan for this experiment consists of:

1. Lectures for about two hundred students in which major mathematical concepts of the course are presented. The following provisions were made:

    A. Adequate facilities for speaking and for display of written work

    B. Adequate seating, lighting, and ventilation.

    C. Adequate time and assistance provided the instructor for preparation of materials needed for large-group instruction

2. Discussion groups of twenty to twenty-five class members under the direction of leaders who were selected on the basis of their knowledge of relevant mathematical concepts as well as their interest in working with elementary school teachers. These group leaders

    A. Provide class members opportunities to raise questions and assist them in answering those questions.

    B. Reinforce basic ideas introduced in the lecture to the total class.

    C. Give individual help when it is needed.

    D. Handle routine details such as checking attendance, administering tests, and distributing materials.

    E. Attend a briefing session before and after each class and also attend each large lecture to the total class.

When one teaches a large class he is often overcome by numbers. It becomes imperative for the instructor to have (a) an adequate supply of texts, reference books, and instructional aids, and (b) adequate assistance for the checking of assignments, texts, and the preparation of materials to be handed out to the class members.

This particular organizational procedure proved effective for this experiment. However, this procedure should be subjected to further experimentation. Additional research is needed regarding the entire problem of class size. Experimentation should be extended to other subject areas as well as other educational levels.

Thus far, research evidence has failed to verify that a "magic number" can be associated with class size.

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APPENDIX

Raw Data
A Test of Mathematical Understanding
A Study of Attitude toward Arithmetic
Evaluation of Course by Students in Large Class
Gains in mathematics shown in relation to gains in attitude for the large Jacksonville class
Gains in mathematics shown in relation to gains in attitude for the Gainesville class
Gains in mathematics shown in relation to gains in attitude for the Yulee class
Gains in mathematics shown in relation to gains in attitude for the Cocoa class
Answer sheet for Test of Understanding of Basic Arithmetic Concepts

RAW DATA

Gainesville - Regular Size Class

Number Correct					
Mathematics Test #1	Diff.	Mathematics Test #2	Average Attitude Score #1	Diff.	Average Attitude Score #2
56	1	57	8.6	.3	8.9
55	5	60	7.8	.1	7.9
52	5	57	7.4	.0	7.4
50	4	54	4.0	-.1	3.9
50	6	56	5.4	3.0	8.4
49	5	54	8.4	-.6	7.8
46	12	58	7.1	.8	7.9
46	9	55	7.8	.0	7.8
45	7	52	7.4	.0	7.4
43	9	52	7.4	.0	7.4
43	12	55	4.9	5.4	8.4
43	11	54	4.3	2.3	6.6
42	14	56	7.4	.0	7.4
40	17	57	7.6	-.6	7.0
38	8	46	3.4	1.3	4.7
37	7	44	6.9	.4	7.3
36	9	45	5.0	2.4	7.4
34	11	45	7.4	.0	7.4
32	4	36	5.8	1.4	7.2

Gainesville - Regular Size Class (Con't.)

Number Correct			Average Attitude Score #1	Diff.	Average Attitude Score #2
Mathematics Test #1	Diff.	Mathematics Test #2			
32	6	38	8.7	.1	8.8
31	15	46	7.4	.4	7.8
31	10	41	8.0	.2	8.2
27	12	39	6.6	2.4	9.0
15	13	28	6.6	-.4	6.2

RAW DATA

Jacksonville - Large MS 300

Number Correct					
Mathematics Test #1	Diff.	Mathematics Test #2	Average Attitude Score #1	Diff.	Average Attitude Score #2
60	5	65	8.0	-.3	7.7
53	5	58	8.8	-.6	8.2
53	6	59	7.1	-.3	6.8
52	6	58	7.8	.5	8.3
50	4	54	8.8	.1	8.9
50	7	57	7.8	.1	7.9
50	9	59	6.9	1.7	8.6
50	14	64	6.1	1.7	7.8
50	13	63	5.4	1.5	6.9
50	6	56	7.8	0	7.8
49	8	57	4.6	-.9	3.7
49	10	59	8.8	-.2	8.6
49	10	59	7.4	.4	7.8
49	7	56	6.4	.7	7.1
48	16	64	7.5	0	7.5
48	9	57	7.5	-.1	7.4
47	12	59	8.9	0	8.9
47	-7	40	7.9	-.5	7.4
47	10	57	5.4	2.0	7.4

Jacksonville - Large MS 300 (Con'd)

Number Correct Mathematics Test #1	Diff.	Mathematics Test #2	Average Attitude Score #1	Diff.	Average Attitude Score #2
46	3	49	8.5	-.3	8.2
46	10	56	9.1	-.6	8.5
46	9	55	2.6	4.8	7.4
46	10	56	3.5	1.5	5.0
46	-4	42	7.6	-.2	7.4
46	7	53	7.6	-2.1	5.4
45	17	62	7.4	1.0	8.4
45	12	57	7.4	-.9	6.8
45	10	55	7.8	-.4	7.4
45	6	51	7.4	1.0	8.4
44	14	58	7.4	.1	7.8
44	3	47	7.8	.5	8.3
44	8	52	7.1	-.5	6.6
44	11	55	6.9	-2.3	4.6
43	14	57	7.4	0	7.4
43	15	58	6.5	1.5	8.0
43	8	51	7.4	0	7.4
43	11	54	5.4	1.5	6.9
43	9	52	6.4	1.8	8.2
42	10	52	7.7	-.2	7.5
42	11	43	9.3	-1.0	8.3

Jacksonville - Large MS 300 (Cont'd)

Number Correct					
Mathematics Test #1	Diff.	Mathematics Test #2	Average Attitude Score #1	Diff.	Average Attitude Score #2
42	12	54	7.4	-.3	7.1
41	11	52	8.0	-.6	7.4
41	13	54	6.7	-.2	6.5
40	12	52	7.9	-.3	7.6
40	9	49	8.6	.3	8.9
40	7	47	8.6	-1.2	7.4
40	11	51	8.1	.2	8.3
40	10	50	7.2	.2	7.4
40	14	54	3.9	4.0	7.9
39	11	50	4.7	4.1	8.8
39	10	49	2.1	3.4	5.5
39	-9	30	7.6	-.2	7.4
39	12	51	7.9	.4	8.3
38	16	54	7.8	-.4	7.4
38	14	52	7.8	.1	7.9
38	7	45	6.8	1.1	7.9
38	10	48	3.3	4.1	7.4
38	12	50	2.8	2.4	5.4
38	17	55	7.4	.4	7.8
38	19	58	2.4	4.5	6.9
38	10	48	2.4	.5	2.9

Jacksonville - Large MS 300 (Cont'd)

Number Correct Mathematics Test #1	Diff.	Mathematics Test #2	Average Attitude Score #1	Diff.	Average Attitude Score #2
38	17	55	5.4	2.0	7.4
37	9	46	7.8	-.6	7.2
37	7	44	9.2	-1.2	8.0
37	11	48	6.2	-.4	5.8
37	12	49	4.3	.6	4.9
37	13	50	7.4	0	7.4
37	16	53	6.6	1.2	7.8
37	9	45	7.5	.3	7.8
37	12	49	6.6	.8	7.4
37	10	47	7.8	.2	8.0
37	11	48	7.4	0	7.4
36	16	52	7.4	-.3	7.1
36	18	54	8.3	.1	8.4
36	9	45	7.4	-1.4	6.0
36	7	43	5.9	1.6	7.5
36	22	58	7.9	.7	8.6
35	18	53	2.4	3.3	5.7
35	20	55	7.4	.9	8.3
35	17	52	4.7	1.0	5.7
35	21	56	8.3	-.2	8.1
35	-2	33	7.8	.3	8.1

Jacksonville - Large MS 300 (Cont'd)

Number Correct Mathematics Test #1	Diff.	Mathematics Test #2	Average Attitude Score #1	Diff.	Average Attitude Score #2
35	20	55	7.1	.3	7.4
35	11	46	9.3	-1.0	8.3
35	16	51	8.2	-.9	7.3
35	-13	22	2.7	3.6	6.3
34	13	47	5.4	2.8	8.2
34	5	39	6.1	.3	6.4
34	12	46	7.4	0	7.4
34	9	43	4.6	0	4.6
34	13	47	7.4	-1.5	5.9
34	9	43	6.8	.7	7.5
34	23	57	6.4	.3	6.7
34	25	59	7.8	1.0	8.8
33	22	55	7.8	-.3	7.5
33	22	55	7.8	.1	7.9
33	0	33	7.4	1.0	8.4
33	20	53	6.2	1.2	7.4
33	10	43	6.4	1.0	7.4
33	19	52	8.0	.6	7.4
33	16	49	7.5	.1	7.6
32	18	50	4.7	.2	4.9
32	16	48	7.2	-.4	6.8

Jacksonville - Large MS 300 (Cont'd)

Number Correct					
Mathematics Test #1	Diff.	Mathematics Test #2	Average Attitude Score #1	Diff.	Average Attitude Score #2
32	8	40	8.5	-3.1	5.4
32	29	61	5.0	2.5	7.5
32	5	37	8.2	-.8	7.4
31	27	58	7.9	.1	8.0
31	10	41	2.6	2.5	5.1
31	16	47	7.4	0	7.4
31	13	44	6.9	-.3	6.6
31	17	48	7.5	-1.4	6.1
31	18	49	7.4	0	7.4
31	8	39	4.9	2.7	7.6
31	12	43	5.8	-1.6	7.4
31	28	59	6.1	3.0	9.1
30	25	55	6.0	1.4	7.4
30	14	44	6.1	1.7	7.8
30	24	54	8.4	-.7	7.7
30	20	50	8.3	-.9	7.4
29	8	37	8.2	.2	8.4
29	14	43	7.9	-.1	7.8
29	13	42	3.8	3.1	6.9
29	19	48	8.4	.3	8.7
29	17	46	8.9	-1.1	7.8

Jacksonville - Large MS 300 (Cont'd)

Number Correct					
Mathematics Test #1	Diff.	Mathematics Test #2	Average Attitude Score #1	Diff.	Average Attitude Score #2
28	19	47	9.1	-.3	8.8
28	25	53	6.4	-.5	5.9
28	21	49	7.4	1.1	8.5
28	26	54	7.4	-.3	7.1
27	22	49	4.9	2.2	7.1
27	16	43	6.6	1.7	8.3
27	10	37	3.1	-.5	2.6
27	22	49	7.4	-.3	7.1
27	20	47	2.4	1.3	3.7
26	-3	23	2.6	0	2.6
26	18	44	6.0	.9	6.9
26	10	36	3.1	3.2	6.3
26	13	39	4.2	-1.3	2.9
26	24	50	7.5	.3	7.9
25	7	32	2.3	1.5	3.8
25	27	52	7.8	.2	8.0
25	22	47	7.5	-1.9	5.6
25	19	44	2.8	2.3	5.1
25	21	46	4.6	1.5	6.1
24	17	41	5.2	.1	5.3

Jacksonville - Large MS 300 (Cont'd)

Number Correct Mathematics Test #1	Diff.	Mathematics Test #2	Average Attitude Score #1	Diff.	Average Attitude Score #2
24	11	35	7.9	-.6	7.3
24	7	31	6.3	1.1	7.4
24	10	34	6.4	1.1	7.5
24	22	46	7.4	0	7.4
24	32	56	8.1	-.5	7.6
24	24	48	5.6	1.1	6.7
23	22	45	6.9	-.3	6.6
22	21	43	3.1	2.9	6.0
22	35	57	5.7	1.3	7.0
22	14	36	7.7	-1.6	6.1
22	9	31	7.4	0	7.4
22	27	49	5.5	2.4	7.9
21	6	27	8.4	-1.0	7.4
21	21	42	5.6	-.2	5.4
21	2	23	7.4	-1.6	5.8
21	15	36	2.3	-.2	2.1
21	23	44	2.5	2.8	5.3
21	14	35	4.6	0	4.6
21	15	36	4.9	.3	5.2
20	28	48	7.4	.4	7.8
20	0	20	3.4	1.7	5.1

Jacksonville - Large MS 300 (Cont'd)

Number Correct					
Mathematics Test #1	Diff.	Mathematics Test #2	Average Attitude Score #1	Diff.	Average Attitude Score #2
20	27	47	8.9	-.3	8.6
19	36	55	8.2	-3.5	4.7
19	16	35	7.8	.1	7.9
19	19	38	4.9	2.5	7.4
19	10	29	6.1	-.4	5.7
19	6	25	4.1	.5	4.6
18	7	25	6.4	.8	7.2
18	35	53	6.0	2.3	8.3
18	9	27	3.8	4.1	7.9
17	6	23	8.5	-.4	8.1
17	22	39	7.5	-.4	7.1
17	36	53	3.1	2.2	5.3
17	11	28	2.4	2.4	4.8
17	18	35	5.2	.1	5.3
17	25	42	7.5	-.1	7.4
16	14	30	5.3	.8	6.1
16	28	44	5.4	1.9	7.3
15	26	41	3.4	3.2	6.6
14	16	30	7.5	.3	7.8
12	20	32	4.2	1.6	5.8
12	29	41	2.3	3.7	6.0

RAW DATA

Cocoa

Number Correct					
Mathematics Test #1	Diff.	Mathematics Test #2	Average Attitude Score #1	Diff.	Average Attitude Score #2
58	7	65	8.8	-1.7	7.1
57	8	65	8.8	-.4	8.4
55	6	61	8.0	.8	8.8
54	5	59	6.3	1.4	7.7
54	4	58	8.4	-.6	7.8
53	6	59	7.5	.1	7.6
53	-4	49	7.8	-.2	7.6
51	0	51	3.8	4.2	8.0
50	7	57	8.4	-.2	8.2
48	9	57	9.1	0	9.1
48	11	59	7.8	-.4	7.4
48	5	53	8.8	-.2	8.6
47	3	50	7.4	.9	8.3
46	8	44	8.8	-.3	8.5
45	7	52	7.3	1.1	8.4
43	13	56	7.4	0	7.4
43	11	44	8.6	-.4	8.2
43	3	46	6.6	-1.2	5.4
42	7	49	6.3	1.1	7.4

Cocoa (Cont'd)

Number Correct					
Mathematics Test #1	Diff.	Mathematics Test #2	Average Attitude Score #1	Diff.	Average Attitude Score #2
42	14	56	6.9	.2	7.1
42	2	46	7.8	-.4	7.4
41	-2	39	8.4	-.2	8.2
39	9	48	6.3	.9	7.2
37	2	39	4.2	.7	4.9
36	11	37	7.6	.6	8.2
31	13	44	7.1	1.0	6.1
30	20	50	8.0	.6	8.6
30	20	50	2.0	.2	2.2
28	23	51	6.6	-.6	6.0
27	14	41	3.8	.8	4.6
27	24	51	6.6	.6	7.2
27	0	27	2.0	.4	2.4
24	23	47	8.1	.3	8.4
22	15	37	8.4	-.8	7.6
21	10	31	4.5	-.7	3.8
19	10	29	2.4	0	2.4

RAW DATA

Yulee

Number Correct			Average Attitude Score #1	Diff.	Average Attitude Score #2
Mathematics Test #1	Diff.	Mathematics Test #2			
52	8	60	7.4	0	7.4
49	6	55	7.5	.5	8.0
49	5	54	7.1	0	7.1
44	12	56	8.1	.8	8.9
43	10	53	6.8	0	6.8
41	-2	39	3.7	.3	3.4
35	13	48	4.4	.9	5.3
34	18	52	7.4	0	7.4
33	9	42	3.5	.2	3.7
33	12	45	6.4	.7	7.1
32	3	35	5.4	.9	6.3
31	8	39	6.2	1.4	7.6
27	0	27	6.3	.3	6.6
27	10	37	5.8	3.0	8.8
27	17	44	5.9	.6	6.5
22	8	30	3.8	.5	4.3
19	17	36	3.5	4.6	8.1
15	13	28	2.8	.9	3.7

A TEST OF MATHEMATICAL UNDERSTANDING

Instructions: Read each question carefully and select the choice you believe to be correct. Mark your answer on the answer sheet provided. Please do not write on the test.

1. How many different single digit numerals are used in our number system?

    a. nine
    b. ten
    c. more than ten but less than twenty
    d. less than nine

2. For each addition fact, two related subtraction facts can be explained. For the addition fact, 6 + 3, which are the two related subtraction facts?

    a. 9 - 6 and 8 - 2
    b. 8 - 2 and 8 - 5
    c. 9 - 3 and 9 - 6
    d. 6 - 3 and 3 - 6

3. In the example of addition given below, the "carry" numbers are circled. Which statement is not true regarding carry numbers?

    Example:

$$\begin{array}{r} 11 \\ 895 \\ \underline{658} \\ 1533 \end{array}$$

    a. One always puts down the larger digit in the sum and "carries" the smaller digit.
    b. These "carry" numbers are obtained by a process of regrouping.
    c. When two single digit numbers are added, the largest possible "carry" number is one.
    d. The (1) above the 7 represents (10).

4. If A and B are any two whole numbers, then the difference of (A - B)

    a. may be a fraction
    b. will always be a whole number
    c. May not be a whole number
    d. is always greater than zero

5. In the subtraction example, which of the following actions might be taken?

Example:
$$\begin{array}{r} 3605 \\ \underline{-\ 1927} \end{array}$$

a. A one is taken from the 6 and placed with the 5 making 15.
b. One thousand must be taken from the 3 thousands and changed into ten hundreds.
c. One ten must be taken from 0 tens leaving nine tens.
d. It would give an incorrect answer if any of the above actions were taken.

6. In the indicated subtraction, if the same number N were added to the minuend and subtrahend, the difference would be:

Example (1)
$$\begin{array}{r} 58 \\ \underline{-\ 39} \end{array}$$

Example (2)
$$\begin{array}{r} 58 + N \\ \underline{-(39 + N)} \end{array}$$

a. The difference in example (2) would be larger than the difference in example (1).
b. The difference in example (1) would be larger than the difference in example (2).
c. Unchanged, therefore these differences would be equal.
d. The differences may be equal for these examples, but this is not generally true.

7. An illustration of the distributative property is:

a. $(5 \times 6) \times 3 = 5 \times (6 + 3)$
b. $5 \times 6 = 6 \times 5$
c. $5 \times (6 + 3) = (5 \times 6) = (5 \times 3)$
d. $(5 + 6) + 3 = 5 + (6 + 3)$

8. The partial products in the multiplication example can best be illustrated by:

Example:
$$\begin{array}{r} 23 \\ \underline{62} \\ 46 \\ \underline{138} \\ 1462 \end{array}$$

a. $62 \times (20 + 3) = (62 \times 20) + (62 \times 3)$
b. $23 \times (2 + 60) = (23 \times 2) + (23 \times 60)$
c. putting 62 down 23 times and adding
d. putting 23 down 62 times and adding

9. In the partial products, the 138 is really a shortcut for:

Example:

$$\begin{array}{r} 23 \\ \underline{62} \\ 46 \\ \underline{138} \\ 1426 \end{array}$$

a. 60 x 23
b. 62 x 3
c. 62 x 23
d. 6 x 23

10. Which statement is not true regarding the indicated multiplicant?

Example:

$$\begin{array}{r} 45 \\ \underline{25} \\ 225 \\ \underline{90} \\ 1125 \end{array}$$

a. The product can be checked by division.
b. The product can be checked by interchanging the factors (putting 25 on top and 45 on the bottom and multiplying again).
c. The product can be checked by counting how many 45's can be subtracted out of 1125.
d. 1125 can represent 45 groups with 45 in each group.

11. The multiplication fact, 7 x 9, could not be indicated by:

a. 7 x (5 + 4)
b. (7 x 10) -1
c. 7 x (6 + 3)
d. (4 + 3) x 9

12. In the indicated division, $62\overline{)22196}$, if the dividend is multiplied by 10, then the quotient would be:

a. ten times larger
b. ten times smaller
c. one hundred times larger
d. none of the above are correct

13. In the indicated division, subtracting 46 from 49 is really a short cut for subtracting:

Example: [see following page]

$$\begin{array}{r} \underline{213} \\ 23\overline{)4913} \\ \underline{-46} \\ 31 \\ \underline{-23} \\ 83 \\ \underline{-69} \\ 14 \end{array}$$

a. 4600 from 4900
b. 46 from 4913
c. 6 from 9 and 4 tens from 4 tens
d. 4600 from 4913

14. In the example, which statement is not true?

Example:

$$\begin{array}{r} \underline{312} \\ 23\overline{)4913} \\ \underline{-46} \\ 31 \\ \underline{-23} \\ 83 \\ \underline{-69} \\ 14 \end{array}$$

a. The dividend = (divisor x quotient) $\pm$ remainder.
b. Two hundred 23's were subtracted in the first round.
c. The remainder after the first subtraction is 313.
d. The dividend ÷ divisor = quotient ÷ remainder.

15. If A and B are whole numbers, one-half of the product, A x B cannot be found by:

a. $\frac{A}{2} \times B$

b. $\frac{A}{2} \times \frac{B}{2}$

c. $\frac{1}{2} \times A \times B$

d. $\frac{A \times B}{2}$

16. Which of the following mathematical properties supports the procedure of changing $1.2\overline{)7.23}$ into $12\overline{)72.3}$ before finding the quotient?

a. For all numbers A, $A \times 1 = A$
b. For all numbers except zero, $\frac{A}{A} = 1$
c. For all numbers A, $A \times 0 = 0$
d. $A \div B$ asks for one to find the number of B's in A

17. For any two whole numbers, A and B, suppose $A \div B = C$, leaving a zero remainder. Then:

a. $3A \div 3B = 3C$
b. $10A \div 10B = C$
c. $A \times C = B$
d. $C \times A = B$

18. Suppose the dividend in the indicated division was written in expanded form, then which statement is not true?

Example (1) $15\overline{)635}$

Example (2) $15\overline{)600 + 30 + 5}$

a. It would be impossible to do division using the form of example (2)
b. If division were carried through in both forms, the quotient would be unchanged.
c. Example (2) could be written as: $\frac{600}{15} + \frac{30}{15} + \frac{5}{15}$.
d. The form in example (1) is the most common.

19. 10.8 million can be written as:

a. 10,000,000.80
b. 18,000,000.00
c. 10,800,000.00
d. 10,080,000.00

20. In the number 148,630, the 4 has the value (with respect to place value):

a. 4 x 100,000
b. 4 x 10,000
c. 100,000
d. 10,000

21. In the number 542,104, the 4 on the left represents a value how many times as large as the 4 on the right?

a. 1,000
b. 10,000
c. 100,000
d. The correct answer is not given above.

22. Approximately how many tens are contained in the number 16502?

    a. 165
    b. 0
    c. 1650
    d. 16502

23. A number can have many names. Another name for the number 2 is:

    a. $\frac{-\sqrt{4}}{-\sqrt{2}}$
    b. 200%
    c. 100% - 98%
    d. None of the above are correct.

24. The number 6571 cannot be written as:

    a. $6 \times 1000) + (5 \times 100) + (7 \times 10) + 1$
    b. $6(10)^3 + 5(10)^2 + 7(10)^1 + 1(10)^0$
    c. $6500 + 70 + 1$
    d. 6.571×10^2

25. Which of the following is not a name for the number 1?

    a. $\frac{.5}{.5}$

    b. $\frac{3}{2} + \frac{2}{3}$

    c. $\frac{2}{3} \times \frac{3}{2}$

    d. $3\frac{1}{2} \times \frac{2}{7}$

26. A specific example of the commutative or reverse law of addition is:

    a. $5 \times 6 = 6 \times 5$
    b. $2 + 3 = 5$
    c. $(2 + 3) + 4 = 2 + (3 + 4)$
    d. $11 + 12 = 12 + 11$

27. Which statement below is not true about zero?

    a. Every number will divide zero, leaving a zero remainder.
    b. Zero may appear in the denominator of a fraction.
    c. If $a \times b \neq 0$, then $a = 0$ or $b = 0$ or $a = b = 0$.
    d. Zero may appear in the numerator of a fraction.

28. Which statement below is not true about the number one?

    a. If a number multiplied by its inverse (reciprocal), the product is 1.
    b. Another name for 1 is 100%.
    c. For any number n, then $n \times 1 = n$ and $n \div 1 = n$
    d. $0 \div 0 = 1$

29. If the whole number 2 and also 3 would both divide some whole number C (leaving a zero remainder), then:

    a. Four would also divide C leaving a zero remainder.
    b. Six would also divide C leaving a zero remainder.
    c. Both four and six would divide C leaving a zero remainder.
    d. None of the above statements are necessarily true.

30. Select the answer below that is not correct for the following statement:
    For any whole number, N, then N is divisible (leaving a zero remainder) by:

    a. Five, if N ends in 0 or 5.
    b. Nine, if the sum of the digits of N is divisible by 9.
    c. Six, if 2 and 3 exactly divide N.
    d. Any even number, if N is even.

31. If a whole number, N, is greater than 1, and the only two whole numbers that will divide (with a zero remainder) N, are 1 and N, then N is called:

    a. an even number
    b. a composite number
    c. an odd number
    d. a prime number

32. If $a + b = 0$ where neither a nor b is zero, then:

    a. $-a = -b$
    b. $-b = -a$
    c. $-b = (-1) \times (-a)$
    d. All of the above statements are true.

33. Which of the following mathematical sentences is an equation? Select the best choice.

    a. $5 + 4 = 9$
    b. $5 \times 4 = 20$
    c. $5y = 20$
    d. All these sentences are equations.

34. Which type of unit is used to express the measure of area?

    a. linear
    b. cubic
    c. square
    d. perimeter

35. If the width of a room is measured, the type units used would be:

    a. square
    b. linear
    c. It would depend on the width of the room.
    d. None of the above statements are correct.

36. The amount of water it would take to fill a given container could be measured in:

    a. linear units
    b. cubic units
    c. It would depend on the size of the container.
    d. It would depend on the shape of the container.

37. If □ , △ , and 4 are different numerals and if □ + △ = □ and □ x 4 = 4 , then

    a. 4 is zero and □ is 1
    b. △ + 4 = 4
    ✓ c. 4 + □ = 4 x □
    d. □ x △ = 4

38. In which answer below is one type (linear, square, cubic) of measurement used in both situations?

    a. Amount of floor space in our classroom, amount of water in our aquarium.
    b. Capacity of my refrigerator, size of Mr. Jones' farm.
    c. Height of the pine tree, amount of window opening in the classroom.
    d. Distance to the moon, perimeter of my bicycle wheel.

39. Which row of numbers is arranged in order from smallest to largest?

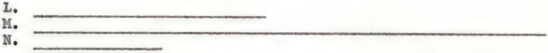
    a. $\frac{3}{14}$, $\frac{3}{7}$, $\frac{3}{10}$, $\frac{3}{3}$

    b. $\frac{4}{16}$, $\frac{4}{2}$, $\frac{4}{3}$, $\frac{4}{4}$

    c. $\frac{5}{16}$, $\frac{5}{12}$, $\frac{5}{10}$, $\frac{5}{8}$

d. $\frac{3}{4}, \frac{3}{5}, \frac{3}{6}, \frac{3}{7}, \frac{3}{8}$

In examples 40 and 41, select the response which seems to be nearest the correct value.



40. The measure of line segment M is what part of that of L?

a. 170%

b. $\frac{37}{14}$

c. $\frac{896}{1053}$

d. $\frac{314}{23}$

41. The measure of line segment N is what part of that of L?

a. $\frac{3}{216}$

b. $\frac{12}{23}$

c. $\frac{8}{43}$

d. $\frac{51}{23}$

42. The number 35.12 cannot be written as:

a. 35 ones + 12 hundredths
b. 3 tens + 5 ones + 1 tenth + 2 hundredths

c. $(35 \times 10) + (12 \times \frac{11}{100})$

d. $(351 \times \frac{1}{10}) + (2 \times \frac{1}{100})$

43. The number 1000 is:

a. 100 times as large as .001
b. 1000 times as large as .001
c. 100,000 times as large as .001
d. 1,000,000 times as large as .001

44. Which statement is false?

a. $\frac{2}{5} + \frac{4}{7} = \frac{4}{10} + \frac{8}{14}$

b. $0 \div \frac{3}{4} = 0$

c. $0 \times \frac{3}{4} = 0$

d. $\frac{1}{3} + \frac{4}{7} = \frac{1}{3} \times 3) + \frac{4}{7} \times 3)$

45. The expression $\frac{4}{5} \times 15$ asks for:

a. The number of $\frac{4}{5}$'s in 15.
b. Four of the five equal parts of 15.
c. Five of the four equal parts of 15.
d. None of the above

46. If zero were added to 15, the result should be:

a. The same as multiplying by ten.
b. 1.5
c. 15
d. .15

47. If any rational number (whole number or fraction) N is divided by $\frac{2}{3}$, which of these statements is always true of the answer?

a. The answer is always greater than N.
b. The answer is always greater than $\frac{2}{3}$
c. The answer will be greater than $\frac{2}{3}$ but less than N.
d. It is impossible to tell unless one knows the exact value of N.

48. How many hundredths of a mile is contained in the entire distance 22.02 miles?

a. 2
b. 200
c. 220
d. 2202

49. Which number is half-way between .5 and .05?

a. .25
b. .025
c. .275
d. None of the above are correct.

50. Which number is one-third of the way from 2.5 to 8.5?

a. 3.0
b. 4.0
c. 4.5
d. None of the above

51. The number .0239 is nearest to which of these numbers?

a. $\frac{1}{8}$
b. $\frac{1}{40}$
c. .017
d. $\frac{1}{10}$

52. One thousand and 100 ten-thousandths is written as:

a. 1000.0100
b. .1100
c. 1000.110
d. 1000.100

53. In the number $.03\frac{1}{3}$

a. the $\frac{1}{3}$ is in the thousandth "position."
b. the $\frac{1}{3}$ means $\frac{1}{3}$ of a hundredth.
c. The decimal point is in the ones position.
d. None of the above statements are correct.

54. The expression $6 \div \frac{1}{3}$ asks for:

a. How many $\frac{1}{3}$'s are in 6.
b. One of the 3 equal parts of 6.
c. Finding the amount of six $\frac{1}{3}$'s.
d. All the above statements are correct.

55. If $\frac{a}{b} x \frac{c}{d} = 1$, then which statement below is not appropriate (true)?

a. These two numbers are reciprocal or inverses of each other.

b. $\frac{a}{b} = \frac{d}{c}$

c. $\frac{a}{b}$ may be less than $\frac{c}{d}$.

d. $\frac{a}{b} = \frac{c}{d}$.

56. Which expression below is not a name for 30%?

a. $\frac{30}{100}$

b. $\frac{3}{10}$

c. .30

d. $\frac{1}{3}$

57. If .001 is multiplied by .01, the answer would be:

a. .0001
b. .01
c. .00001
d. None of the above are correct.

58. If $\frac{a}{b} = \frac{c}{d}$, then (where a,b,c, and d are not zero):

a. $a \times c = b \times d$
b. $a \times b = c \times d$
c. $a \times d = b \times c$
d. It is impossible to tell unless the exact value of a, b, c, and d are given.

59. In the indicated division: $3.12/\overline{66.35}$, if the divisor were divided by 10 and the dividend were divided by 10, then:

a. The indicated quotient would be unchanged.
b. The indicated quotient would be ten times less.
c. The indicated quotient would be ten times greater.
d. The indicated quotient would be 100 times less.

60. The number of times $\frac{2}{3}$ is contained in one whole unit is:

a. 1
b. $1\frac{1}{3}$
c. $1\frac{1}{4}$
d. $1\frac{1}{2}$

61. If a whole number is divided by a proper fraction (a value less than 1):

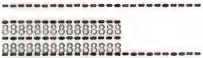
    a. The quotient will always be a mixed number.
    b. The quotient will always be larger than the dividend.
    c. The divisor will be larger than the dividend.
    d. None of the above statements are necessarily correct.

62. If the numerator of a fraction is multiplied by some whole number, A, the value of the fraction:

    a. Is multiplied by A.
    b. Is divided by A.
    c. May be affected in any of the above ways.
    d. It is impossible to tell unless one knows the exact value of A.

63. The shaded portion of the rectangle represents what part of the rectangle?

    Example:



    a. $\frac{2}{3}$ of $\frac{3}{5}$

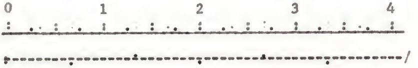
    b. $\frac{3}{2}$ of $\frac{3}{5}$

    c. $\frac{6}{9}$ of $\frac{2}{5}$

    d. None of the above.

64. The drawing below could best represent which of the following:

    Example:



    a. $3 - \frac{3}{4} - 1\frac{1}{4}$

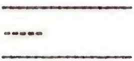
    b. $3 \times 1\frac{1}{4}$

    c. $1\frac{1}{4} \times 3$

    d. $3 - \frac{3}{4} + 3$

65. The shaded portion of the drawing would best represent which of the following:

Example:



a. $\frac{1}{2} \times \frac{1}{3}$

b. $3 \times \frac{1}{6}$

c. $\frac{1}{3} \times \frac{1}{2}$

d. $\frac{1}{6} \times 3$

66. An experienced teacher might use the drawing below to illustrate:

Example:

000 00000
000 00000
000 00000
000 00000
000 00000

a. 15 + 25

b. 5(3 + 5) = (5 x 3) + (5 x 5)

c. $\frac{1}{5}$ of 25

d. $\frac{1}{3}$ of 15

67. When we divide by a common fraction, we invert the divisor and multiply. Which statement best tells why we invert?

a. It is merely a rule that needs no explanation.
b. It is a quick method for finding common denominators for the divisor and dividend.
c. The product of the reciprocal of the divisor and the dividend will give the same answer as the dividend divided by the divisor.
d. The dividend multiplied by one over the divisor, reciprocal, is the same as dividing the dividend by the divisor.

A STUDY OF ATTITUDE TOWARD ARITHMETIC

W. H. Dutton
University of California, Los Angeles   Revised Form C, Scale 5, 1964

Name ________________ Male ___ Female ___ Date of test ____________19___

Read the statements below. Choose statements which show your feelings toward arithmetic. Let your experiences with this subject in the elementary school determine the marking of items.

Place a check (v) before those statements which tell how you feel about arithmetic. Select only the items which express your true feelings --probably not more than five items.

3.2 1. I avoid arithmetic because I am not very good with figures.
8.2 2. Arithmetic is very interesting.
2.0 3. I am afraid of doing word problems.
2.5 4. I have always been afraid of arithmetic.
8.7 5. Working with numbers is fun.
1.0 6. I would rather do anything else than do arithmetic.
7.7 7. I like arithmetic because it is practical
1.5 8. I have never liked arithmetic.
3.7 9. I don't feel sure of myself in arithmetic.
7.0 10. Sometimes I enjoy the challenge presented by an arithmetic problem.
5.2 11. I am completely indifferent to arithmetic.
9.5 12. I think about arithmetic problems outside of school and like to work them out.
10.5 13. Arithmetic thrills me and I like it better than any other subject.
5.6 14. I like arithmetic but I like other subjects just as well.
9.8 15. I never get tired of working with numbers.
16. Place a circle around one number to show how you feel about arithmetic in general.
1 2 3 4 5 6 7 8 9 10 11
Dislike Like
17. My feelings toward arithmetic were developed in grades:
1 2 3 4 5 6 7 8 9 10 11 other _____ (Circle one)
18. My average grades made in arithmetic were: A B C D (Circle one)
19. List two things you like about arithmetic.
a.
b.
20. List two things you dislike about arithmetic.
a.
b.

LARGE CLASS EVALUATION FORM

1. How significant do you consider this course?
No Value _____ Some Value _____ Extremely Valuable _____

2. Have "new ideas" been presented that were useful in your classroom?
None _____ Some: _____ Many _____

3. To what extent has your interest been aroused?
Little or no interest _____ Some interest _____ Extremely interested _____

4. Have the small class discussions been helpful?
No _____ Somewhat helpful _____ Very helpful _____

5. Has the pace of the instructor (large class) been satisfactory?
Too slow _____ Just right _____ Little too fast _____ Too fast __

6. To what extent has the content and ideas been "New" to you?
None new _____ Some new _____ Mostly new _____

7. Have you felt free to ask questions in the small classes?
No hesitancy _____ Some hesitancy _____ Great hesitancy _____

8. Do you have difficulty in knowing what to read or study or what is expected of you?
No difficulty ______ Some difficulty _____ Lot of difficulty _____

9. How meaningful have the illustrations and explanations been to you?
No value _____ Some were meaningful _____ Very meaningful ______

10. Would you consider it worthwhile for other teachers to take this course?
Not worthwhile _____ Worthwhile to a few _____ Extremely worthwhile _____

11. If a sequel to this course (MS 305) were offered at a later date, would you be interested in taking that course?
No _____ Possibly _____ Yes _____

12. How well have you understood the content presented?
None of it _____ Some of content _____ Most of content _____

Gains in Mathematics	Gains in Attitude: less than 0	0-.09	1.0-1.9	2.0-2/9	3.0-3.9	4.0 -	F
35-39	1		1	2			4
30-34	1						1
25-29	4	3	3	2	3		15
20-24	9	7	6	3			25
15-19	8	16	5	3	1	1	34
10-14	23	15	8	5	3	3	57
5- 9	14	14	7	2		2	39
0- 4	2	1	1				4
less than 0	2	3	1		1		7
F	64	59	32	17	8	6	186

Gains in mathematics shown in relation to gains in attitude for the large Jacksonville class

	Gains in Attitude						
Gains in Mathematics	less than 0	0-.9	1.0-1.9	2.0-2.9	3.0-3.9	4.0 -	F
35-39							
30-34							
25-29							
20-24							
15-19		2					2
10-14	1	4		2		1	8
5- 9	1	7	1	1	1		11
0- 4	1	1	1				3
less than 0							
F	3	14	2	3	1	1	24

Gains in mathematics shown in relation to gains in attitude for the Gainesville class

	Gains in Attitude						
Gains in Mathematics	less than 0	0-.9	1.0-1.9	2.0-2.9	3.0-3.9	4.0 -	F
35-39							
30-34							
25-29							
20-24							
15-19		2				1	3
10-14		5		1			6
5- 9		5	1				6
0- 4		2					2
less than 0		1					1
F		15	1	1		1	18

Gains in mathematics shown in relation to gains in attitude for the Yulee class

	Gains in Attitude						
Gains in Mathematics	less than 0	0-.9	1.0-1.9	2.0-2.9	3.0-3.9	4.0 -	F
35-39							
30-34							
25-29							
20-24	1	4					5
15-19	1						1
10-14	3	5	1				9
5- 9	5	4	3				12
0- 4	3	2				1	6
less than 0	2	1					3
F	15	16	4			1	36

Gains in mathematics shown in relation to gains in attitude for the Cocoa class

ANSWER SHEET FOR TEST OF UNDERSTANDING OF BASIC ARITHMETIC CONCEPTS

YOUR NAME ______________________________

Please place an X on your choice of answers for each of the corresponding questions.

1. a b c d
2. a b c d
3. a b c d
4. a b c d
5. a b c d
6. a b c d
7. a b c d
8. a b c d
9. a b c d
10. a b c d
11. a b c d
12. a b c d
13. a b c d
14. a b c d
15. a b c d
16. a b c d
17. a b c d
18. a b c d
19. a b c d
20. a b c d
21. a b c d
22. a b c d
23. a b c d
24. a b c d
25. a b c d
26. a b c d
27. a b c d
28. a b c d
29. a b c d
30. a b c d
31. a b c d
32. a b c d
33. a b c d
34. a b c d
35. a b c d
36. a b c d
37. a b c d
38. a b c d
39. a b c d
40. a b c d
41. a b c d
42. a b c d
43. a b c d
44. a b c d
45. a b c d
46. a b c d
47. a b c d
48. a b c d
49. a b c d
50. a b c d
51. a b c d
52. a b c d
53. a b c d
54. a b c d
55. a b c d
56. a b c d
57. a b c d
58. a b c d
59. a b c d
60. a b c d
61. a b c d
62. a b c d
63. a b c d
64. a b c d
65. a b c d
66. a b c d
67. a b c d

BIOGRAPHICAL SKETCH

Jackie Lee Foley was born November 26, 1930, in Lockhart, Alabama. He began his undergraduate work at Middle Georgia College, Cochran, Georgia. He transferred to the University of Florida, Gainesville, Florida, where he received his Bachelor of Science in Education degree. He received his Master of Education degree from the University of Florida in 1960.

While pursuing his doctoral program at the University of Florida, he has been primarily employed by the University. The summer of 1963 and 1964 were spent teaching at the University of Texas.

Membership is held in Phi Kappa Phi, Phi Delta Kappa, The National Council of Teachers of Mathematics, and the Florida Council of Teachers of Mathematics.

This dissertation was prepared under the direction of the chairman of the candidate's supervisory committee and has been approved by all members of that committee. It was submitted to the Dean of the College of Education and to the Graduate Council, and was approved as partial fulfillment of the requirements for the degree of Doctor of Education.

Kimball Wiles
Dean, College of Education

Dean, Graduate School

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